It's About Time: Implications of the Period Length in an Equilibrium Search Model

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Abstract

Empirical evidence suggests that transitions between employment states are highly clustered around the first day of each workweek or month. I analyze the effect of this phenomenon by presenting an equilibrium search model in which the period length is a parameter determining the degree of clustering. Infinitesimally short periods result in a continuous-time model with bilateral meetings, while longer time periods introduce the possibility of recall or simultaneity of job offers. In this environment, I show that the period length has a profound effect on equilibrium outcomes, including the unemployment rate, unemployment duration, and the cross-sectional wage distribution.

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1 Introduction

1.1 Motivation and Summary

Modern labor markets are characterized by large flows between employment states. For example, the Bureau of Labor Statistics (BLS) reports that of all U.S. workers who were employed in August 2012, 1.3 percent was unemployed one month later. In the same month, 19.3 percent of the unemployed workers made the transition to employment. Similar patterns can be observed for other months or other countries. Given their importance for a large variety of labor market outcomes, understanding these flows and their implications is a key objective of the economic literature.[2]

This paper will argue that one aspect of the flows has been neglected so far. Due to the monthly nature of most data, including the BLS data, little is known about the exact timing of transitions between employment states within a month. Often, it is simply assumed that the transitions take place uniformly across time. Using detailed administrative data on the begin and end dates of nearly 700,000 unemployment spells in France, I show that this assumption is questionable. Instead, the data indicate a high degree of *clustering*. That is, a disproportionate fraction of the monthly transitions takes place on a few specific days, in particular on the first day of the month and the first day of the workweek (Monday). To be precise, 33 percent of all unemployment spells end on the first day of the month, and the likelihood of a spell ending on a Monday is 2 times larger than on a Tuesday, 3 times larger than on a Friday, and 8 times larger than on a Sunday. Similar clustering can be observed for separations.

The aim of this paper is to analyze the implications of these observations for the labor market equilibrium. Existing models of labor market flows cannot directly be used for this goal, since they generally specify a structure which does not allow for comparative statics with respect to the degree of clustering.[3] I therefore present an alternative, i.e., an on-the-job search model that allows for varying degrees of clustering in a tractable way. This is achieved by making the model's period length a parameter which can take arbitrary values; transitions between employment states will be clustered if the period length is positive, while convergence to a continuous-time model without clustering is obtained if the period length approaches zero.

In this environment, workers randomly meet with firms making wage offers. A change in the period length affects both the likelihood that a worker meets multiple firms in the same period and the wage offers that these firms make. As a result, I find that the period length (i.e., the degree of clustering) has profound effects on equilibrium outcomes. First, a change in the period length affects the monthly flows between unemployment and employment. A longer period leads to a higher unemployment rate and longer unemployment durations, since it worsens the frictions in the model, even if the underlying rate at which workers and firms meet remains unchanged. Further, the period length affects the cross-sectional wage distribution (the 'earnings distribution') and wage mobility patterns.

[2] Matching and separation rates affect, e.g., (un)employment rates, income distributions, and incentives to invest in human or physical capital.

[3] See the literature review in section 1.2 for a more detailed discussion.

To be precise, when the period length is positive, the equilibrium gives rise to wage dispersion in the form of a wage ladder. That is, workers will earn a relatively low wage in their first job after an unemployment spell and must make one or more job-to-job transitions before they can obtain higher wages. The speed at which workers climb this ladder – as measured by the wage gain between two subsequent jobs – is stochastic and the resulting earnings density is non-monotonic. As I will discuss in section 5, all of these properties are consistent with empirical evidence, but none of them emerge in the continuous-time limit of the model and only a subset is generated by existing models.

These results are important for a variety of reasons. For example, data on monthly flows and the unemployment rate are often used to calibrate the rate at which workers and firms match and/or separate. Since the duration of a period affects the relation between these variables, such an exercise (implicitly) requires an assumption about the right value of the period length. Although these assumptions are rarely stated or motivated, they can bias the calibration. Similarly, economists frequently use wage data to identify certain elements of their model, such as productivity distributions or the flow payoff of unemployed workers. Again, misspecification of the period length or the wage formation process may lead to wrong inference.[4]

Having provided a rough summary, I now describe the model and results in greater detail. In the model, time is fundamentally a continuous variable and workers and firms randomly meet each other according to a Poisson process. However, they cannot immediately start production upon meeting; they must first wait a certain amount of time instead. Such an assumption is rather natural and can – for example – be motivated by the idea that frictions in the matching process typically make it difficult to perfectly align the timing of the meeting with the emergence of demand for the worker's services; consider, for example, a teacher who is needed at exactly the start of the new school year, a worker who needs to execute a task in a particular phase of a group project, or a worker who needs to replace a current employee leaving the firm at the end of the month. As long as the time points at which production can commence and acceptance decisions must be made ('transition moments') are correlated across firms, clustering arises.[5]

When meeting a worker, a firm makes a job offer. In line with recent empirical evidence by Barron et al. (2006) and Hall and Krueger (2012), the offered wage has a take-it-or-leave-it (TIOLI) character and is conditional on the worker's current wage, but does not depend on outside offers.[6] The delay between the meeting and the start of production creates incentives for workers to not immediately accept a firm's job offer, but to keep it in consideration while simultaneously continue searching for potentially better job offers. Firms can generally not force workers to immediately sign a contract and abandon their search, since they cannot credibly threaten to withdraw the job offer in such a situation.

[4] See e.g. Shimer (2005, 2012) for a calibration of matching rates and Bontemps et al. (1999, 2000) for an estimation of productivity distributions.

[5] Clustering in separations can be introduced in a similar way by assuming that workers and firms must give each other an advance notice of termination of the match.

[6] See section 2 for a more detailed discussion of the empirical evidence regarding wage formation and its implementation in the model.

Consequently, workers can recall recent offers and may be able to choose between multiple offers. This possibility must be taken into account by firms when making job offers. The degree of clustering determines the likelihood of simultaneous offers and will therefore affect labor market outcomes.

To keep the model as simple as possible, I assume that the transition moments are perfectly correlated across firms.[7] This effectively creates a discrete-time model in which the amount of time between two transition moments – i.e., the period length – is a parameter that determines the degree of clustering. As mentioned before, the amount of clustering diminishes if the period length decreases. In the limit, if the period length becomes infinitesimally short, the model converges to a continuous-time setup in which transitions are spread uniformly across time.

In this environment, I characterize the equilibrium and derive its properties, most of which are quite intuitive. With a positive period length, firms making wage offers face uncertainty about the number of other poaching firms that are competing for the same worker. This uncertainty causes firms to use a mixed strategy in determining their wage offers, as first shown by Burdett and Judd (1983).[8] Hence, wage dispersion is a fundamental characteristic of the model. Since firms can condition their strategy on the current wage of the worker that they try to hire, not one but a large number (formally, a continuum) of overlapping wage offer distributions arise in equilibrium. Workers start an employment spell at a low wage, but then climb the wage ladder through job-to-job transitions until they are hit by a job destruction shock. The wage increase between two jobs depends on the realized draw from the relevant wage offer distribution and is therefore stochastic. Relatively few workers earn very high wages, because those require many job-to-job transitions without intermittent unemployment spell, which is an unlikely event. Similarly, very few workers experience prolonged employment at very low wages. As a result, most workers earn intermediate wages, making the earnings density non-monotonic.

When the period length converges to zero, the uncertainty in the degree of competition that a poaching firm faces disappears; firms will realize that the probability that no other poaching firm is trying to hire the same worker at the same moment goes to 1. This eliminates the need for firms to randomize their wage offers. Instead, they will simply offer slightly more than the worker's current wage, creating an equilibrium in which the earnings distribution is degenerate at the workers' flow value of unemployment, as in Diamond (1971). Hence, the period length is an important determinant of equilibrium outcomes.

After deriving the properties of the equilibrium, I compare them to empirical evidence and the predictions of existing models. Not surprisingly, the data strongly favor the model with clustering over the model without clustering. More interestingly, existing models generally also fail to generate one or more of the equilibrium properties of the model with clustering, as I will explain in detail in section 5.

[7] See section 2.2 for a discussion of this assumption.

[8] Burdett and Judd (1983) consider a model with buyers and sellers. See Gautier and Moraga-Gonzalez (2004) and Gautier et al. (2007) for labor market versions of their model.

This paper proceeds as follows. Section 2 presents the empirical observations that motivate the research question and various modeling assumptions. The model is introduced in section 3 and section 4 derives the equilibrium. In section 5, I describe some of the positive implications of the model, analyze how they vary with the period length, and compare them to the literature. Section 6 provides a discussion and section 7 concludes.

1.2 Relation to Literature

This paper adds to the literature on search models of the labor market in a couple of ways. First and foremost, the paper deals with time in the model in a novel way. In the search literature, two approaches regarding the specification of time can be distinguished. Papers in the spirit of Mortensen and Pissarides (1994), Moen (1997), Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002a,b) explore continuous-time models, while papers like Delacroix and Shi (2006), Gautier et al. (2007), Menzio and Shi (2010, 2011) and Wolthoff (2010) analyze discrete-time papers. None of these models is directly suitable to study the effects of clustering on the labor market equilibrium. After all, transitions between employment states are spread uniformly across time by assumption in the steady state of continuous-time models, while all existing discrete-time models use an exogenously given, fixed period length. By allowing for arbitrary period lengths, the model in this paper forms a natural intermediate case between both approaches, which can be used to analyze various degrees of clustering.

Second, this paper merges the literatures on simultaneous search and on-the-job search. The idea that simultaneity or recall of job offers can change equilibrium outcomes has been explored by various authors, including Burdett and Judd (1983), Galenianos and Kircher (2009), Kircher (2009), and Carrillo-Tudela et al. (2011).[9] However, these models do not allow for search on the job and therefore fail to generate the wage mobility patterns observed in the data. On the other hand, models of on-the-job search, such as Burdett and Mortensen (1998), Postel-Vinay and Robin (2002a,b), Delacroix and Shi (2006) and Menzio and Shi (2010, 2011), assume that workers meet with at most one poaching firm at a time and rely on different model features to avoid the Diamond paradox. The exact approach is different in each of the papers, but always causes the models' predictions to differ from the empirical observations that my model matches, as I will discuss in detail in section 5.

Third, my paper contributes by analyzing a model in which firms make TIOLI offers that are customized to the workers' current wage. A recent survey by Hall and Krueger (2012) suggests that this is an adequate description of many firms' behavior, as I will discuss in section 2. Nevertheless, this contract space appears to be novel in the literature. For example, wage posting models in the spirit of Burdett and Mortensen (1998) capture the TIOLI nature of the offers, but do not allow for customization; a firm may fail to hire a worker by making an offer below his current wage. This

[9] See also Eeckhout and Kircher (2010), who study how a seller's optimal strategy depends on whether he can meet multiple buyers simultaneously or not.

implies not only that some gains from trade are left on the table, but also that all wages can be reached directly from unemployment and that the cross-sectional wage density is monotonically increasing.[10] The customization of the wage offers relates my model to work by Carrillo-Tudela (2009a,b), who allows poaching firms to observe the employment status and/or experience of workers, but not the contract. As an intermediate case between my model and Burdett and Mortensen (1998), he obtains a wage ladder with just two 'rungs', one for unemployed workers and one for employed workers. Finally, the fact that firms know workers' current wages implies that my model shares some features with the directed search literature, in which workers with different wages sort themselves into different submarkets, see e.g. Delacroix and Shi (2006) and Menzio and Shi (2010, 2011). I will discuss this connection in more detail in section 5.4.

2 Empirical Motivation

This section discusses a number of empirical facts motivating the research question of this paper and the contract space explored in the model.

2.1 Empirical Observations

Clustering of Employment Transitions The key empirical observation motivating this paper is the high degree of clustering of employment transitions. While data on the begin and end dates of employment relationships is generally scarce, precise administrative data on the begin and end dates of unemployment spells is available in the French national register of unemployed workers (*Fichier National des Assédic*). For my analysis, I use count data on the number of unemployment spells ending or beginning at a particular combination of a date and a day of the week, based on a 2.5 percent random sample of all 27.7 million spells occurring between 1990 and 2003.[11]

I then regress the logarithm of the count on date and weekday dummies. The results of this exercise are reported in table 1 and 2 for end dates and begin dates, respectively.[12] The estimated coefficients are displayed as *incidence-rate ratios* relative to the base categories (first day of the month for dates; Monday for weekdays). Hence, a value of 0.5 for a particular day means that – on average – half as many transitions take place on that day as on a typical Monday.

The null hypothesis of no clustering is overwhelmingly rejected in both tables. Instead, two striking patterns emerge. First, most transitions take place on the first day of each month. Even though that day constitutes only 3 percent of calendar time, it captures 33 percent of the ending spells and 13 percent of the starting spells. Smaller (but statistically significant) peaks are also visible at the 15th and

[10] See Mortensen (2003) and Galenianos and Kircher (2009) for a discussion of the latter property.

[11] I thank Gregory Jolivet for providing these numbers. See Ferracci et al. (2010) for a detailed description of the data. Similar data is unfortunately not readily available for the U.S..

[12] A negative binomial model gives virtually identical results. A Poisson model is rejected because of overdispersion in the data.

the end of each month. Second, in a typical week, Mondays show significantly more transitions than any other day. The difference is especially large compared to Sundays (8 times as many for ending spells and 12 times as many for beginning spells), but even compared to e.g. Fridays, the difference is a factor 2 (beginning spells) or 3 (ending spells). Jointly, the date and weekday dummies explain the vast majority (approximately 95 percent) of the variation in the data.

Of course, unemployment spells may begin or end for other reasons than transitions to / from employment and some degree of measurement error might be present in the data. Nevertheless, considering the strength of the patterns and the administrative character of the data, these results strongly suggest a tendency for transitions to take place at the beginning of the workweek and/or the beginning of the month.

Customized Wage Offers A second empirical observation is the prevalence of wage offers that seem to be customized to a worker's individual circumstances. In a recent paper, Hall and Krueger (2012) discuss empirical evidence on wage formation obtained from a representative survey among U.S. workers in 2008. One set of results concerns the question how well employers know the outside options of the workers that they try to hire. In the sample, approximately half of the respondents indicates that their current or most recent employer learned how much they were making in their previous job before making the job offer. This fraction is slightly higher for recent job losers (66 percent), but otherwise reasonably constant across different categories of workers, ranging between 41 percent for blue-collar workers to 53 percent for women.[13] As Hall and Krueger argue, employers presumably request this information to customize their offer to the outside option of the worker, i.e., to make sure that their offer is satisfactory.

Take-it-or-Leave-it Offers A third empirical observation concerns the negotiability of the wage offers. In the survey by Hall and Krueger (2012), 63 percent of the respondents indicate that the wage offer that they received had a TIOLI character and that no bargaining took place. Interestingly, little difference exists between workers who could have kept an existing job and workers who could not; the fraction of those workers indicating TIOLI offers is 60 percent and 65 percent, respectively.

As Hall and Krueger (2012) remark, these sample averages are likely underestimates of the true prevalence of TIOLI offers since the unit of observation is a worker instead of a hiring event, and the numbers are higher for younger and less-educated workers, who experience more turnover. For example, they report TIOLI offers for 94 percent of the individuals classified as blue-collar workers. These results are supported by evidence from a survey by Krueger and Mueller (2011) among Unemployment Insurance (UI) recipients in New Jersey in the fall of 2009 and the spring of 2010. In that sample, three quarters of all job offers have a TIOLI character.[14]

[13]These results are not included in the published version of their paper, but can be found in the web appendix.

[14]As Hall and Krueger (2012) discuss, some care is required in the interpretation of these numbers since the fact that a worker *did not* make a counteroffer does not necessarily imply that he *could not* make one. Nevertheless, it seems safe to conclude that TIOLI offers are common in many labor markets.

Counteroffers A last empirical observation concerns firms' responses to outside offers. Barron et al. (2006) study this topic using data from the Small Business Administration survey among U.S. employers in 2001. In one of the questions, employers were asked to indicate whether they would consider to raise a worker's wage in order to retain him as an employee, in case he received a job offer from another company. In 53 percent of the cases, counteroffers would not be made. Some variation across sectors exists, with the fraction of negative responses ranging from 33 percent in mining and agriculture to 71 percent in transport. In general however, it seems that firms frequently abstain from counteroffers.

2.2 Interpretation and Implementation

As described in the introduction, I interpret the clustering in the transitions as evidence that when a worker and a firm meet, they cannot always start production immediately. Frictions in the matching process may make it hard to perfectly align the timing of the meeting with the emergence of demand for the worker's services, implying that the worker and the firm may need to wait for some time before production can start. This waiting period creates an incentive for the worker to not immediately accept a firm's job offer upon meeting, but to simultaneously keep searching for potentially better job offers. As a result, a situation may arise in which the worker can choose between multiple offers. This simultaneity is the key difference compared to continuous-time models in which meetings are bilateral and agents either match immediately or go separate ways.

To keep the model tractable, two simplifying assumptions are made. First, as mentioned before, I assume that the transition moments are perfectly correlated across firms, effectively resulting in a discrete-time model in which the period length is a parameter. While this assumption may appear strong, since it rules out matching or separation between two transitions moments, it is in fact mostly technical in the sense that the key mechanism, i.e., the potential simultaneity of job offers, survives with a less-than-perfect correlation. Second, I assume – as standard in the search literature – that workers can recall offers within a period, but not across periods.

In line with the second empirical observation, I will allow firms to make job offers that are conditional on the current employment state and wage of the worker. This is essentially a relaxation of the assumption in Burdett and Mortensen (1998) that such customization is not possible. In reality, firms have a strong incentive to make sure that their wage offers are acceptable, because failing to do so implies that they forgo their share of the rents created by the meeting.

Following the third and fourth empirical observation, I assume that firms do not respond to offers by (other) poaching firms. Note that although the last empirical observation (no counteroffers) originates in a different source, it is entirely consistent with the evidence from the survey by Hall and Krueger (2012). After all, if an outside offer were to create a bidding war between the current employer and the poaching firm (as in Postel-Vinay and Robin, 2002a,b), the poaching firm would neither be making TIOLI offers nor have a strong incentive to learn the worker's current wage before submitting a first offer. While it may not seem subgame perfect to ignore offers by other firms, it is straightforward to think of reasons why a large number of firms in the data use such a strategy. First, matching outside offers is not necessarily an optimal strategy from an ex ante point of view, since it provides workers with stronger incentives to search for outside offers if search intensity is endogenous (see e.g. Postel-Vinay and Robin, 2004; Moscarini, 2005). Second, while a worker's current employment state and wage can easily be verified[15], it might be complicated to verify the validity of offers from (potentially unknown) poaching firms that have no incentive to facilitate this verification (see Mortensen, 2003, for a detailed discussion).

3 Model

3.1 Environment

Agents Consider the steady state in a labor market populated by a measure 1 of homogeneous workers and a positive measure of homogeneous firms. Time is denoted by t and is continuous. All agents are infinitely-lived and risk-neutral. Firms employ workers in order to produce output according to a production technology that exhibits constant returns to scale. Each worker can supply one indivisible unit of labor and is therefore at a given moment in time either employed at one of the firms or unemployed. I denote the measures of employed and unemployed workers by $1-u$ and u respectively.

Payoffs Both firms and workers discount future payoffs at rate $\widehat{\rho}$. A worker who is employed by a firm produces a flow output $\widehat{y}$. From this output, the firm pays the worker a wage $\widehat{w}$ at each instant for as long as the match lasts. Unemployed workers have a flow payoff equal to $\widehat{h}$, consisting of unemployment benefits, home production, and their value of leisure. In order to guarantee the existence of a market, assume that $\widehat{h}$ is strictly smaller than $\widehat{y}$.

Transitions Workers and firms meet each other according to a Poisson process. When a meeting takes place, the firm makes a take-it-or-leave-it wage offer. As common in the literature, I allow the job offer arrival rate to differ across employment states; unemployed workers receive job offers at rate λ_U, while employed workers meet new firms at rate λ_E. To keep notation as simple as possible, assume $\lambda_E \leq \lambda_U$. Each job offer consists of a wage that the firm promises to pay to the worker for the entire duration of the match. Jobs are subject to negative shocks leading to job destruction at a rate δ. As discussed above, transitions between employment states are only possible at certain predetermined time points. I assume that these moments occur every $\tau \geq 0$ units of time, i.e., at $t \in \{\tau, 2\tau, 3\tau, ...\}$, creating discrete time periods of length τ; if τ goes to zero, convergence to a continuous-time model is obtained.

[15] For example, by calling references or asking for a copy of a salary statement.

Discrete-Time Representation In the remainder of this paper, I will simplify notation by using the discrete-time equivalents of the model's parameters when possible. I denote the periodical discount factor by $\rho(\tau) = e^{-\widehat{\rho}\tau}$. The discounted value of periodical output equals $y(\tau) = \beta(\tau)\widehat{y}$, where

$$\beta(\tau) = \int_0^\tau e^{-\widehat{\rho}t} dt = \frac{1}{\widehat{\rho}}\left(1 - e^{-\widehat{\rho}\tau}\right).$$

Likewise, I define $w(\tau) = \beta(\tau)\widehat{w}$ and $h(\tau) = \beta(\tau)\widehat{h}$. Time aggregation also needs to be taken into account for the transition rates, since workers may receive multiple job offers per period. To be precise, the number of firms that a worker with employment state $s \in \{U, E\}$ (U = unemployment, E = employment) meets in any given period follows a Poisson distribution with mean $\lambda_s\tau$. I denote the probability that the worker obtains j job offers by $p_s(j|\tau) = e^{-\lambda_s\tau}\frac{(\lambda_s\tau)^j}{j!}$. Likewise, the number of job destruction shocks in a period follows a Poisson distribution with mean $\delta\tau$.

Offers and Earnings As discussed above, a firm may condition its wage offer x on the worker's current wage w (or on the fact that he is unemployed), but not on the offers of other poaching firms. Let $F(x|w)$ be the equilibrium distribution of wage offers x to workers earning w and $F(x|h)$ the distribution of wage offers to the unemployed.[16] The corresponding supports are $\mathcal{F}(w)$ and $\mathcal{F}(h)$ respectively. In equilibrium, these distributions will imply a steady state earnings distribution $G(w)$, representing the fraction of employed workers earning a wage lower than w.[17] Denote the corresponding density by $g(w)$ and its support by $\mathcal{G}$. Sometimes I will use expressions that apply to both employed and unemployed workers. For this purpose, define $\mathcal{G}' = \mathcal{G} \cup \{h\}$.

Matching At the end of the period, each worker accepts the best job offer that he has received, as long as it gives him a higher value than staying in his current job (or staying unemployed). Let $r(w)$ be the reservation wage of a worker currently earning w. I denote the workers' matching probability by $\mu_s(\tau)$. An expression for $\mu_s(\tau)$ will be derived in the next section.

Separation The end of the period is also the moment at which job destruction takes places. While jobs are subject to negative shocks in continuous time, I again assume – for simplicity but in line with table 2 – that unemployment spells can only start at one of the transition moments. A match is terminated if the job was hit by at least one negative shock in the current period. Hence, the periodical job destruction probability equals $\phi(\tau) = 1 - e^{-\delta\tau}$.

Note that for $\tau > 0$, an assumption is required on whether workers can lose their new job immediately after (i.e., in the same period as) finding it. I will assume that this is the case, and that

[16] Although unemployed workers technically do not earn a salary, I will in the remainder of this paper often simplify notation by treating them as workers earning h.

[17] I characterize the earnings distribution in section 5.3.

the probability that this happens is independent of the moment at which the worker and the firm first met.[18]

3.2 Value Functions

To facilitate the equilibrium derivation, I specify the workers' and firms' Bellman equations. Note that since no transitions are possible within a period, attention can be restricted to the time points τ, 2τ, 3τ, etc.

Workers' Value Functions Consider the workers' value functions first. Let V_U denote a the value of unemployment and $V_E(w)$ the value of being employed at a wage $w \in \mathcal{G}$ at the beginning of a period. During the period, each worker receives a flow payoff, either from employment or from household production. The discounted values of these payoffs are w and h respectively. An unemployed worker gets $j \in \mathbb{N}_0$ job offers from the distribution $F(\cdot|h)$ with probability $p_U(j)$. He accepts the best wage offer as long as the associated payoff is higher than the payoff of remaining unemployed. After matching, job destruction may occur in which case the worker flows back to unemployment. So, the expected value of an offer x equals[19]

$$\widetilde{V}_E(x) = (1-\phi) V_E(x) + \phi V_U.$$

The value of unemployment is then equal to

$$V_U = h + \rho \sum_{j=1}^{\infty} p_U(j) \int_{x \in \mathcal{F}(h)} \max\left\{\widetilde{V}_E(x), V_U\right\} dF^j(x|h) + \rho p_U(0) V_U. \tag{1}$$

A similar expression holds for the value of employment. The worker gets $j \in \mathbb{N}_0$ job offers with probability $p_E(j)$. Again, the worker accepts the best offer, but only if it gives a higher payoff than rejecting it. Hence, $V_E(w)$ equals

$$V_E(w) = w + \rho \sum_{j=1}^{\infty} p_E(j) \int_{x \in \mathcal{F}(w)} \max\left\{\widetilde{V}_E(x), \widetilde{V}_E(w)\right\} dF^j(x|w) + \rho p_E(0) \widetilde{V}_E(w). \tag{2}$$

Formally, the strategy of workers can be described by acceptance sets, consisting of the wage

[18]This assumption can be motivated by assuming that each firm has a continuum of job openings. Upon meeting, a worker has to be assigned to a particular job opening and this choice is irreversible. Negative shocks hit both filled and unfilled jobs, but only cause job destruction after signing contracts at the end of the period.

The reason for this assumption is technical. Alternative assumptions imply that workers are willing to accept wage cuts, solely because their current job can be destroyed while a new job cannot (or only with a lower probability). This seems an unrealistic and undesirable feature, not in the last place because the magnitude of the wage cut that workers are willing to accept would vary with the period length. Assuming that workers can lose their job immediately after finding it avoids this issue without qualitatively affecting the key mechanisms in this paper. Note further that δ will be small in any reasonable calibration, such that immediate job loss will be rare.

[19]To simplify notation, I will suppress the dependence of parameters on the period length τ when no confusion is possible.

offers that they are willing to accept given their current wage. A worker rejects all wage offers that are not part of his acceptance set. Given monotonicity of the value functions, each acceptance set can be characterized by a reservation wage. A worker currently earning $w \in \mathcal{G}'$ accepts his best wage offer x if it is higher than his reservation wage $r(w)$ and rejects it otherwise.[20]

Firms' Value Functions Next, consider the firms' Bellman equations. Firms may employ multiple workers, but the payoff of hiring a specific worker is independent of the number or employment conditions of other workers in the firm, since the production function exhibits constant returns to scale. I therefore specify the value functions per individual job. Specifically, denote a firm's value of employing a worker at wage w by $V_F(w)$. The firm gets an instantaneous payoff of $y - w$. The match continues in the next period if the worker does not make a job-to-job transition and no job destruction shock takes place. In equilibrium, any new wage offer will always be higher than the current wage of a worker, since firms can condition their wage offers on the current wage. Hence, the worker only does not make a job-to-job transition if he does not meet any new firm. The firm's value of a job therefore equals

$$V_F(w) = y - w + \rho p_E(0)(1-\phi)V_F(w). \tag{3}$$

A firm that meets a worker currently earning $w \in \mathcal{G}'$ and makes a job offer x obtains a positive payoff if it manages to hire the worker. Let $m(x|w)$ denote the firm's hiring probability. The firm faces a trade-off. Offering a higher wage increases the matching probability $m(w|x)$, since workers can compare offers. However, it simultaneously lowers the value $V_F(w)$ of the future match. Firms will offer wages such that they maximize

$$V_V(x|w) = \rho m(x|w)(1-\phi)V_F(x). \tag{4}$$

3.3 Equilibrium Definition

The equilibrium can now be defined. In equilibrium, workers and firms must optimally choose their reservation wages and wage offers, respectively. Note that wage offers weakly below h or equal to y can never be part of an equilibrium, since they provide a firm with a zero payoff. The support of the earnings distribution will therefore be an endogenously determined subset of the interval (h, y). Again treating unemployed as earning h, the strategy of a worker or a firm must then be specified for each $w \in [h, y)$. Hence, taking the definitions of the value functions (1), (2) and (4) as given, I formally define the equilibrium as follows.

Definition 1. A steady state market equilibrium ('equilibrium') is a tuple $\left\{u, \{F(\cdot|w), r(w)\}_{w\in[h,y)}\right\}$ such that

[20] Workers that get an offer equal to their reservation wage are indifferent between accepting and rejecting the offer. As a tie-breaking rule I assume that workers always reject in this event of measure zero. This assumption does not affect any conclusions, but simplifies notation because under the opposite assumption both unemployed and employed workers could have a periodical income equal to h.

1. Profit maximization: $V_V(x|w) = \max_{x'} V_V(x'|w)$ for all $x \in \mathcal{F}(w)$, and for all $w \in [h, y)$.

2. Optimal reservation wage: $\begin{cases} V_E(x) = V_U & \text{iff } x = r(h) \\ V_E(x) = V_E(w) & \text{iff } x = r(w), \text{ for all } w \in (h, y). \end{cases}$

3. Steady state: u consistent with labor market flows.

After solving for the equilibrium in the next section, the earnings distribution $G(w)$ will be formally derived in section 5. Note already that its support must be equal to the union of the supports of all equilibrium wage offer distributions, i.e., $\mathcal{G}$ must satisfy

$$\mathcal{G} = \bigcup_{w \in \mathcal{G} \cup \{h\}} \mathcal{F}(w). \tag{5}$$

4 Equilibrium

Firms' Wage Setting Consider a firm which meets a worker currently earning w and which has to decide what wage to offer. The worker will reject wage offers that do not exceed $r(w)$, so $m(x|w) = 0$ for $x \leq r(w)$. However, wage offers in the interval $(r(w), y)$ may be acceptable to the worker. The firm faces a trade-off in this interval: a higher wage offer lowers the future per-period profit, but is more likely to be accepted by the candidate. The likelihood of acceptance depends on the wages that other firms offer. To be precise, the worker will accept the firm's wage offer if it is higher than all his $j-1$ competing wage offers. If a firm offers the job to a worker, the conditional probability that he has $j-1$ other job offers equals $\frac{j p_s(j)}{\sum_{n=1}^{\infty} n p_s(n)} = \frac{j p_s(j)}{\lambda_s \tau} = e^{-\lambda_s \tau} \frac{(\lambda_s \tau)^{j-1}}{(j-1)!}$. For $x > r(w)$, $m(x|w)$ is therefore equal to

$$\begin{aligned} m(x|w) &= \sum_{j=1}^{\infty} e^{-\lambda_s \tau} \frac{(\lambda_s \tau)^{j-1}}{(j-1)!} F^{j-1}(x|w) \\ &= e^{-\lambda_s \tau (1 - F(x|w))}. \end{aligned} \tag{6}$$

Hence, the number of other, better offers follows a Poisson distribution with mean equal to $\lambda_s \tau (1 - F(x|w))$.

Substituting (6) and the solution to (3) into (4) gives the following expression for the expected discounted payoff for a firm offering $x > r(w)$ to a worker earning w:

$$V_V(x|w) = e^{-\lambda_s \tau (1 - F(x|w))} \frac{\rho e^{-\delta \tau} (y - x)}{1 - \rho p_E(0) e^{-\delta \tau}}.$$

The firm maximizes this expression with respect to x, taking the distribution of wage offers by other firms as given. First, I show that there must be wage dispersion in equilibrium and that $F(x|w)$ must be continuous with connected support.

Lemma 1. *In any market equilibrium, for any $w \in [h,y)$ such that $r(w) < y$, the support $\mathcal{F}(w)$ of the wage offer distribution is not a singleton but a convex set of positive measure. Further, the wage offer distribution $F(x|w)$ itself is continuous in x.*

This result, which dates back to Burdett and Judd (1983), is well-known; firms must randomize their wage offers since they do not know whether they face competition from other poaching firms, in which case they want to offer high wages, or not, in which case a wage just above the reservation wage is optimal.

Naturally, firms must be indifferent between all wages in the support of $F(x|w)$. Using this indifference condition, one can derive the candidate wage offer distribution presented in the following lemma.

Lemma 2. *In any market equilibrium, for any $w \in [h,y)$ such that $r(w) < y$, firms post prices according to*

$$F(x|w) = \begin{cases} 0 & \text{if } x \leq r(w) \\ \frac{1}{\lambda_s \tau} \log\left(\frac{y-r(w)}{y-x}\right) & \text{if } x \in \mathcal{F}(w) = (r(w), \bar{x}(w)] \\ 1 & \text{if } x > \bar{x}(w), \end{cases} \tag{7}$$

where $\bar{x}(w)$ equals

$$\bar{x}(w) = e^{-\lambda_s \tau} r(w) + \left(1 - e^{-\lambda_s \tau}\right) y. \tag{8}$$

Workers' Reservation Wage Now consider workers' strategies. Workers' main decision is whether or not to accept their best wage offer, given their current wage or payoff from unemployment. It is straightforward that the workers' value of employment $V_E(w)$ is strictly increasing in w. This implies that workers follow a reservation wage strategy. The reservation wage $r(w)$ is defined by the reservation wage property

$$\begin{cases} V_U = V_E(r(h)) \\ V_E(w) = V_E(r(w)) \quad \text{for all } w \in (h,y). \end{cases} \tag{9}$$

It is immediate that $r(w) = w$ for all $w \in (h,y)$, meaning that employed workers accept all wage offers higher than their current wage. Unemployed workers, on the other hand, take into account that if they accept a job, their job offer arrival rate might change. The following lemma shows that – in equilibrium – their reservation wage must be a weighted average of h and y.

Lemma 3. *In any market equilibrium, the workers' reservation wage function is given by*

$$r(w) = \begin{cases} \frac{h+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)y}{1+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)} & \text{for } w = h \\ w & \text{for all } w \in (h,y), \end{cases} \tag{10}$$

where

$$\gamma = \frac{1}{1-\rho(1-\phi)(1-\Phi_E)}. \tag{11}$$

and

$$\Phi_s = 1 - e^{-\lambda_s \tau} - \lambda_s \tau e^{-\lambda_s \tau}.$$

It is straightforward to check that $r(h) > h$ if and only if $\lambda_U > \lambda_E$. In other words, workers are choosy when their job offer arrival rate will fall after accepting a job. If $\lambda_U = \lambda_E$, this motivation disappears and $r(h) = h$. This result is analogous to Burdett and Mortensen (1998).

Existence and Uniqueness After characterizing the above necessary conditions for the firms' wage offer distribution and the workers' reservation wage function, we can now turn to equilibrium. The following proposition formally establishes that a unique steady state equilibrium exists.

Proposition 1. *A unique steady state equilibrium exists. In this equilibrium, firms' wage offers are determined by lemma 2, while workers' reservation wages are determined by lemma 3. The unemployment rate is equal to*

$$u = \frac{\phi}{\phi + \mu_U}.$$

5 Positive Implications

After deriving the equilibrium, I now describe a number of positive implications of the model and I discuss how they differ from the implications of existing models.

5.1 Wage Dispersion

Full Support First, I consider the support of the earnings distribution $G(w)$, which is implicitly determined by equation (5). From the previous discussion, it immediately follows that the infimum of the support must equal $r(h)$. Since $\mathcal{F}(w) = (w, \bar{x}(w)]$ for all $w > h$, the supremum of $\mathcal{G}$ is determined by the solution to $\bar{x}(w) = w$, which is $w = y$. Hence, the support of the earnings distribution ranges from the reservation wage of an unemployed worker up to the productivity level, i.e., $\mathcal{G} = (r(h), y)$.

Comparison with Literature This feature of the equilibrium contrasts with most other on-the-job search models with homogeneous agents. For example, the support of the earnings distribution is an interval in Burdett and Mortensen (1998), but with an upper bound that is strictly smaller than the productivity level. The main explanation for this difference is the following. In Burdett and Mortensen (1998), all wages in the support must provide firms with the same expected payoff, greater than zero. Given bounded matching rates, this implies that wages sufficiently close to the productivity level cannot be part of an equilibrium.[21] In my model, on the other hand, the indifference condition

[21] The same is true in wage-tenure models such as Burdett and Coles (2003) and Shi (2009).

does not apply to all wages in the support of the earnings distribution. Upon meeting a worker with a good outside option, firms are willing to accept a lower payoff than what other firms are earning, as long as this payoff is positive. Hence, if necessary, firms are willing to offer wages up to the point where surplus becomes equal to zero.

My model shares this feature with e.g. Julien et al. (2000) and Postel-Vinay and Robin (2002a,b), in which Bertrand competition between firms can drive the wage to the productivity level. However, without heterogeneity, the support of the earnings distribution consists of two mass points only in those papers; a low wage for workers whose outside option was unemployment and a wage equal to y for workers who were in touch with more than one firm. Mass points also appear in directed search models with wage posting, like Delacroix and Shi (2006) and Menzio and Shi (2010). In those models, each mass point is the optimal way to attract searchers with a particular outside option, as I discuss in more detail below.

5.2 Wage Mobility

Wage Ladder A second important feature of the equilibrium is that not all wage levels can be reached directly from unemployment. In general, a worker currently earning w cannot earn more than $\overline{x}(w)$ in his next job. Hence, a worker will earn at most $\overline{x}(h)$ in his first job after an unemployment spell, and not more than $\overline{x}(\overline{x}(h))$ in his second job after unemployment. By induction, the maximum wage $\hat{w}_{U,n}$ a worker can earn in his n^{th} job after unemployment is equal to

$$\hat{w}_{U,n} = \overline{x}^{n}(h) \equiv y - e^{-\lambda_U \tau - \lambda_E \tau (n-1)} (y - r(h)) \text{ for } n \in \mathbb{N}_1.$$

Hence, the wage distribution can be seen as a wage ladder; a worker who currently earns a wage w must first find a job in $(w, \overline{x}(w)]$ before he can earn a wage exceeding $\overline{x}(w)$.[22]

Comparison with Literature It is well established in the literature that workers coming out of unemployment earn lower wages than the general population of workers. For example, Jolivet et al. (2006) documents that the wage distribution of all workers first-order stochastically dominates the distribution of entry wages in eleven different countries.[23] By itself, this fact is not necessarily inconsistent with models such as Burdett and Mortensen (1998), in which all wages can be reached directly from unemployment. However, Buchinsky and Hunt (1999) provide more direct evidence for the existence of a wage ladder by estimating the transition probabilities between the quintiles of the wage distribution. They find that most wage mobility takes place either within a quintile or from one quintile to an adjacent quintile.

[22] Note that this is a necessary condition, but not a sufficient one. After finding a job paying $w' \in (w, \overline{x}(w)]$, the worker might move to a job paying a wage w'' satisfying $w' < w'' \leq \overline{x}(w)$. Note further that the definition of a wage ladder used here differs from the one sometimes used to describe the Burdett and Mortensen (1998) model.

[23] Arulampalam (2001), Farber (1999), Gregory and Jukes (2001) and Kletzer (1998) also report lower wages after unemployment.

Nevertheless, equilibrium search models featuring a wage ladder are rare. As discussed above, a homogeneous-agent version of Postel-Vinay and Robin (2002a,b) generates a ladder with two rungs: workers start an employment spell at a low wage, but jump to y as soon as they get an outside offer. In Carrillo-Tudela (2009a), firms observe the employment status (but not the wage) of each worker that they meet and offer a low wage to unemployed workers and higher wages to employed workers.

Only directed search models with on-the-job search yield a larger number of rungs. For example, Delacroix and Shi (2006), Shi (2009), and Menzio and Shi (2010, 2011) all obtain equilibria in which a worker starts an employment spell in a job that provides a low payoff, but subsequently chooses to only apply to jobs yielding higher payoffs. Once that job is obtained, the worker will start applying to an even better job, etc. Note that in these models the speed with which workers climb the ladder is fixed (i.e., workers earning the same wage before a transition to a new job will continue to all earn the same wage after the transition). My model allows for variation in this speed. In line with empirical evidence, some workers experience larger wage increases between two jobs than others.

5.3 Earnings Distribution

Third, I consider the model's implications for the shape of the earnings distribution. After deriving a recursive expression for the distribution, I will show that its density is non-monotonic, i.e., increasing at low wages and decreasing at high wages.

Inflow and Outflow To derive the earnings distribution, I use an approach which is based on worker flows. Define $\mathcal{Z}(w)$ as the set of employed workers earning a wage lower than w, such that – by definition – the probability that a worker is included in this set is given by $G(w)$. We can then exploit the fact that inflow into and outflow from $\mathcal{Z}(w)$ must be equal in steady state to determine $G(w)$.

Note that inflow can only occur from unemployment, since workers never move to jobs paying a lower wage. An unemployed worker flows into $\mathcal{Z}(w)$ if his best wage offer is lower than w and if he is not immediately hit by the job destruction shock. Hence, inflow into the set of workers earning less than w, denoted by $I(w)$, equals

$$I(w) = u(1-\phi)\sum_{j=1}^{\infty} p_U(j) F^j(w|h). \tag{12}$$

On the other hand, outflow from $\mathcal{Z}(w)$ can occur for two reasons. First, all employed workers, irrespective of whether they make a job-to-job transition, are subject to a job destruction shock with probability ϕ in which case they flow back to unemployment. Second, workers may move to a better paying job if they get job offer paying more than w and are not hit by the shock. So, outflow $O(w)$

equals

$$O(w) = (1-u)\int_{r(h)}^{w}\left(\phi + (1-\phi)\sum_{j=1}^{\infty} p_E(j)\left(1-F^j(w|z)\right)\right) g(z)\,dz. \tag{13}$$

Equating (12) and (13) yields an integral equation, which can be solved to obtain the earnings distribution $G(w)$ and the corresponding density $g(w)$.

Cutoff Points Unfortunately, the integral equation cannot be solved for the entire interval $(r(h), y)$ at once, since the support of each $F(w|z)$ is a only strict subset of this interval. I therefore divide $(r(h), y)$ into a number of subintervals by specifying a number of cutoff points $\{\hat{w}_0, \hat{w}_1, \hat{w}_2, \ldots\}$. These cutoff points satisfy $r(h) \equiv \hat{w}_0 < \hat{w}_1 < \ldots < \hat{w}_\infty \equiv y$ and are chosen in such a way that – for each subinterval $(\hat{w}_{n-1}, \hat{w}_n]$ – inflow and outflow, and consequently $G(w)$, can be determined in a straightforward way.

To derive the cutoff points, consider what happens to inflow $I(w)$ and outflow $O(w)$ when we keep increasing w, starting from $r(h)$. Initially, the flows can be calculated after substituting the relevant wage offer distributions into (12) and (13). However, once w is at least equal to $y - e^{-\lambda_E \tau}(y - r(h))$, the workers who were employed at the lowest equilibrium wage can no longer leave the set $\mathcal{Z}(w)$ through a job-to-job transition. Mathematically, the wage offer distribution for these workers is non-differentiable in this point, implying a potential non-differentiability in $G(w)$. Hence, $\hat{w}_{E,1} = y - e^{-\lambda_E \tau}(y - r(h))$ is the first cutoff point. Note that the shape of $G(\cdot)$ around $\hat{w}_{E,1}$ matters when determining the outflow around $\bar{x}(\hat{w}_{E,1})$, which in turns affects outflow around $\bar{x}(\bar{x}(\hat{w}_{E,1}))$, etc. In other words, a 'ripple effect' arises, creating cutoff points at

$$\hat{w}_{E,n} \equiv y - e^{-\lambda_E \tau n}(y - r(h)) \text{ for } n \in \mathbb{N}_1.$$

A second set of cutoff points is due to inflow. Unemployed workers cannot earn more than $\hat{w}_{U,1}$ in their first job in an employment spell. This creates a non-differentiability and thus a cutoff in this point. A similar ripple effect as before implies cutoff points at all $\hat{w}_{U,n}$ for $n \in \mathbb{N}_1$.

Next, combine both sets of cutoff points and let $\hat{w}_n$ be the n^{th} order statistic of the new set, i.e., the n^{th} smallest value. Note that if $\lambda_U > \lambda_E$ then $\hat{w}_{U,1} > \hat{w}_{E,1}$, such that $\hat{w}_1 = \hat{w}_{E,1}$. Let $\hat{n} = \left\lfloor \frac{\lambda_U}{\lambda_E} \right\rfloor$ such that $\hat{w}_{E,\hat{n}}$ is the largest $\hat{w}_{E,n}$ smaller than $\hat{w}_{U,1}$. Then it is straightforward to show that the cutoff points $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ alternate from $\hat{w}_{U,1}$ onwards. Hence, we can write

$$\{\hat{w}_0, \hat{w}_1, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} = \{r(h), \hat{w}_{E,1}, , \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\}.$$

Figure 1 illustrates these cutoff points for a typical choice of parameter values.

Recursive Characterization After deriving the cutoff points, the earnings distribution can be determined. To facilitate this approach, denote the solution for the earnings distribution on the subinterval $(\hat{w}_{n-1}, \hat{w}_n]$ by $G_n(w)$. One can then establish that the earnings distribution has a recursive structure: a closed-form expression can be obtained for $G_1(w)$; knowledge of $G_{n-1}(w)$ is sufficient to derive $G_n(w)$ for all $n \in \{2, \ldots, \hat{n}+1\}$; and knowledge of $G_{n-1}(w)$ and $G_{n-2}(w)$ is sufficient to derive $G_n(w)$ for all $n \in \{\hat{n}+2, \ldots\}$. The following lemma provides a full characterization. The details of the derivation are again relegated to the appendix.

Lemma 4. *In market equilibrium, the earnings distribution is characterized by the following recursive system*

$$\begin{cases} G_1(w) &=& \frac{\phi}{\mu_U}(1-\phi-\mu_U)\left(\left(\frac{y-r(h)}{y-w}\right)^{\Delta_E}-1\right) \\ G_n(w) &=& C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) + (1-\phi)\left(G_{n-1}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int(y-w)^{\Delta_E}\, dG_{n-1}(\underline{w}(w))\right) \\ && \text{if } n \in \{2, \ldots, \hat{n}+1\} \\ G_n(w) &=& C_n + \phi + (1-\phi)\left(G_{n-2}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int(y-w)^{\Delta_E}\, dG_{n-2}(\underline{w}(w))\right) \\ && \text{if } n \in \{\hat{n}+2, \ldots\}, \end{cases}$$

where $\hat{n} = \left\lfloor \frac{\lambda_U}{\lambda_E} \right\rfloor$, $\Delta_E = \frac{1}{\phi+\mu_E}$, $\underline{w}(w) = y - e^{\lambda_E \tau}(y-w)$, *and* C_n *is determined by* $\lim_{w \to \hat{w}_{n-1}} G_n(w) = G_{n-1}(\hat{w}_{n-1})$.

Non-Monotonicity of the Wage Density Lemma 4 implicitly also fully characterizes the earnings density $g(w)$. While the complexity of $G_n(w)$ increases rapidly in n, which impedes the derivation of analytical expressions, one can show that this earnings density is non-monotonic, as formalized in the following lemma.

Lemma 5. *For all* $0 < \tau < \infty$, *the earnings density* $g(w)$ *is 1) increasing for* w *sufficiently close to* $r(h)$, *and 2) decreasing for* w *sufficiently close to* y.

The intuition for this non-monotonicity is as follows. In order to be employed at a low wage, a worker must have gotten a low wage offer after an unemployment spell and have remained there since that moment. On the other hand, in order to earn a really high salary, the worker must have experienced many consecutive job-to-job transitions without a job destruction shock in between. The probability of both events is relatively small and therefore the equilibrium fractions of workers earning these wages are small. Intermediate wage levels are much more common, because there are several ways in which one can obtain such a salary. Some workers get this wage directly after unemployment, while others experience a couple of job-to-job transitions before finding a job paying this wage.

Comparison with Literature Even though non-monotonic earnings densities are in line with empirical evidence, very few equilibrium search models generate them without allowing for ex-ante heterogeneity. Clearly, the earnings density is necessarily monotonic in models in which the support only consists of two mass points, such as the homogeneous version of Postel-Vinay and Robin (2002a,b). However, even wage-posting models with on-the-job search typically imply earnings densities that are either strictly increasing or strictly decreasing. The former is true for e.g. Burdett and Mortensen (1998), while the latter holds for e.g. Delacroix and Shi (2006) and Menzio and Shi (2010).[24]

A few authors have extended the Burdett and Mortensen (1998) model in ways that generate more realistic earnings distributions. For example, Mortensen (2000) shows that adding match-specific capital can help to generate a non-monotonic shape. Burdett et al. (2011) and Fu (2011) explore a model with human capital accumulation, which leads to the long tail typically observed in the data.

5.4 Effect of the Period Length

Lastly, I consider the effect of the period length τ on the equilibrium outcomes. I start by considering the limit $\tau \to 0$, in which case the model converges to a continuous-time model without recall. The effect of larger period lengths will be analyzed by calibrating the model.

Continuous-Time Limit If $\tau \to 0$, each period becomes infinitesimally short. In each period, a worker then receives either zero or one job offer, so there are never multiple firms competing to hire the same worker. This implies that each wage offer will be equal to the reservation wage of the worker in question. Hence, we end up in the Diamond (1971) equilibrium and get an earnings distribution that is degenerate at the level of household production h. Workers flow from unemployment to employment at rate λ_U and flow back at rate δ, such that the unemployment rate equals $\frac{\delta}{\delta+\lambda_U}$. These results are summarized in the following corollary.

Corollary 1. *If $\tau \to 0$, the reservation wage function, the offer distribution and the earnings distribution converge to*

$$r(w) = w, \text{ for all } w \in \mathcal{G}'.$$

$$F(x|w) = \begin{cases} 0 & \text{if } x < w \\ 1 & \text{if } x \geq w, \text{ for all } w \in \mathcal{G}'. \end{cases}$$

$$G(w) = \begin{cases} 0 & \text{if } w < h \\ 1 & \text{if } w \geq h \end{cases}$$

At the beginning of each period, the equilibrium unemployment rate equals

$$u = \frac{\delta}{\delta + \lambda_U}.$$

[24] Some wage-posting models without on-the-job search also generate wage dispersion. For example, the earnings density is increasing in Gautier and Moraga-Gonzalez (2004) and decreasing (given reasonable parameter values) in Galenianos and Kircher (2009). Gautier et al. (2007) generate a non-monotonic density by introducing heterogeneity in search costs.

Hence, the shape of the earnings distribution crucially depends on the period length. Since the earnings distribution is rather complex, it is not straightforward to analyze this relationship analytically. I therefore illustrate the effect of the period length by calibrating the model.

Calibration For the baseline calibration, I set the period length equal one month, $\tau = 1$, in line with the dominant patterns in table 1 and 2. This period length also corresponds with the frequency of the employment data provided by the U.S. Bureau of Labor Statistics, such that there are no complications arising from time aggregation. Following a similar approach as Shimer (2005, 2012), I then find an average monthly job finding probability $\mu_U(\tau)$ of 0.443 and a monthly job destruction probability $\phi(\tau)$ of 0.024 between January 1994 and December 2004.[25] These values imply $\lambda_U = 0.605$ and $\delta = 0.024$. In line with the estimates by Moscarini and Vella (2008), I set the monthly job-to-job transition probability $\mu_E(\tau)$ equal to 0.032, which corresponds to $\lambda_E = 0.033$. I further set $\widehat{h} = 0.4$, $\widehat{y} = 1$ and $\widehat{\rho} = 0.05/12$.

Changes in the Period Length Taking these values as given, one can then calculate the equilibrium outcomes for arbitrary values of τ. The results of this exercise are presented in table 3 for τ between 0 (continuous time) and 12 (annual periods). The bold values represent the baseline calibration with $\tau = 1$. The results confirm that the period length affects both the flows and the wages in equilibrium. For example, a decrease in the period length reduces both the unemployment rate and the expected unemployment duration $D(\tau) = t/\mu_U(\tau)$, even though the underlying rate at which workers and firms meet does not change. Further, the workers' reservation wage, the average wage and the degree of wage dispersion all vary with τ. A longer period length is associated with higher wages, since it increases competition among firms. The degree of wage dispersion as measured by the standard deviation is non-monotonic, since the wage distribution becomes degenerate for both extremely short and extremely long periods. The last column reports the mean-min ratio, a measure of wage dispersion introduced by Hornstein et al. (2011). In the class of models that they consider, this statistic is independent of the period length. This no longer holds in the model presented here: an increase in the period length increases the distance between the average and the lowest wage in the market.[26]

Changes in the Job Offer Arrival Rates For comparison, I also calculate the effects of changes in the job offer arrival rates, while keeping the period length at $\tau = 1$. I choose the alternative values for λ_U and λ_E in such a way that workers' matching probabilities $\mu_U(\tau)$ and $\mu_E(\tau)$ are equal to the corresponding values in table 3. Table 4 displays the results and reveals some striking differences. For example, a decrease in the job offer arrival rates *increases* the search frictions, while a reduction in the period length *decreases* the frictions, as is reflected in their opposing effects on the unemployment

[25] Note that Shimer (2005, 2012) needed to control for time aggregation since he assumed a continuous-time model, i.e., $\tau = 0$. Obviously if $0 < \tau < 1$, a different correction for time aggregation is required.

[26] Note that like the models considered by Hornstein et al. (2011), my model generates a rather small value for the Mm-ratio.

rate and unemployment duration. Further, starting from the baseline scenario $\tau = 1$, $\lambda_U = 0.605$ and $\lambda_E = 0.033$, a change in the period length has a smaller effect on the workers' reservation wage than a corresponding change in the job offer arrival rate. Naturally, the main reason for these differences is that both exercises are fundamentally different: a change in the period length affects the job finding probabilities $\mu_s(\tau)$, the job destruction probability $\phi(\tau)$ as well as the discount factor $\rho(\tau)$, whereas a change in the job offer arrival rates only affects $\mu_s(\tau)$.

Comparison to Literature It is instructive to briefly consider how a change in the period length would change equilibrium outcomes in other models. While time aggregation generally stays important with respect to the flows, the effects on wages are not always as large as in my model. For example, the nature of the earnings distribution would not change in a model like Postel-Vinay and Robin (2002a,b): irrespective of the period length, firms would continue to offer low wages to unemployed workers and engage in Bertrand competition when necessary.

Nevertheless, the effects of the period length hold much more generally than just in the environment that I consider. In the continuous-time model of Burdett and Mortensen (1998), poaching firms know that they only need to beat a random draw from the earnings distribution in order to hire a worker. However, when positive period lengths are introduced, these firms will also need to make sure that their offer beats other poaching firms, and the likelihood of their presence depends on τ.

An interesting similarity also exists between my model and models of directed search on the job, in which firms post (and commit to) contracts that are observed by all workers before they decide where to apply. To see this, consider first the on-the-job search models of Delacroix and Shi (2006) and Menzio and Shi (2010, 2011). Although these models are set up in discrete time, workers send – by assumption – only one application per period. As mentioned, the equilibrium wage distribution consists of a number of mass points $w_0 < w_1 < \ldots w_n$ and workers endogenously sort themselves. That is, unemployed workers exclusively apply to w_0; after obtaining such a job, they start to apply to w_1 instead, etc. Next, consider the (static) multiple-application model of Kircher (2009). This model also gives rise to a number of mass points, $w^1 < w^2 < \ldots < w^m$, where superscript i indicates the i-th simultaneous application.

Combining both models leads to a world in which unemployed workers apply to wages $w_0^1 < w_0^2 < \ldots < w_0^m$, and a worker who is employed at a wage w_j^i applies to wages $w_{j,i}^1 < w_{j,i}^2 < \ldots < w_{j,i}^m$. While the derivation of the equilibrium is not trivial, it follows immediately that the equilibrium gives rise to a wage ladder which workers climb at a stochastic speed. Further, the earnings distribution depends on the period length, since that determines the distribution of the number of simultaneous applications that workers send. Numerical simulation indicates that, for reasonable parameters values, both very low and very high wages are less common in the cross-section than intermediate wages.

Note that because of the commitment and the endogenous separation of workers, a model like this is consistent with the empirical evidence in section 2, and therefore – along certain dimensions – similar to the model presented in this paper. This suggests that the results in this paper are not just an artifact of my model, but are instead more robust properties of models that allow for both simultaneous and on-the-job search in a world in which firms make TIOLI offers or post wages.

6 Discussion

After deriving the decentralized equilibrium and analyzing the effect of a change in the period length, I now briefly discuss the relevance of the results for empirical inference, along with a potentially interesting extension.

6.1 Empirical Inference

The result that equilibrium outcomes vary with the length of a period has potentially important implications for the estimation or calibration of labor market parameters. I discuss a few examples. First, matching and separation rates are often calibrated from data on monthly flows between unemployment and employment (see e.g. Shimer, 2005, 2012). Second, the flow payoff of an unemployed worker, which plays an important role in analyzing the empirical content of job search models[27], has been calibrated from the division of surplus (see Hagedorn and Manovskii, 2008). Third, a popular way to estimate productivity distributions is from wage data, after controlling for frictional wage dispersion on the basis of an equilibrium search model, a technique explored by e.g. Bontemps et al. (1999, 2000) and Postel-Vinay and Robin (2002b). As table 3 revealed, the results of these empirical exercises will crucially depend on the period length that is (implicitly) assumed; misspecification of the period length will generate incorrect estimates.

6.2 Heterogeneity

The model presented in this paper assumed throughout that workers and firms are homogeneous to highlight the effect of the period length in the simplest possible setting. In reality, workers and firms of course exhibit a large degree of heterogeneity, which needs to be taken into account in any more comprehensive empirical analysis. A stylized but particularly tractable way is to assume that the labor market is segmented into a number of homogeneous submarkets, as is done in – for example – van den Berg and Ridder (1998) and Wolthoff (2010).

A more involved alternative is to model how heterogeneous agents interact in the same market. The working paper version of this article explores such a setup with firms differing in their productivity (see Wolthoff, 2011). The period length continues to have important effects on the equilibrium outcomes under this extension. It is shown that within a time period, high-productivity firms always offer higher wages than low-productivity firms. However, the customization of the wage offers and the absence of counteroffers immediately implies that job-to-job transitions might be inefficient in the sense that a worker who currently works for a high-productivity firm may move to a low-productivity firm in

[27] see e.g. Shimer (2005); Hornstein et al. (2011).

the next period (if he did not receive job offers from any other high-productivity firms).[28] Efficiency requires that workers' reservation values are sufficiently high to guarantee that such transitions do not take place, which suggests a new potential role for policy instruments such as unemployment benefits or a minimum wage.

7 Conclusion

Motivated by the observation that transitions between employment states appear to be highly clustered in time, this paper presents a novel equilibrium model of search on the job, which allows for arbitrary period lengths. If the period length is positive, transitions are clustered, but if the period length goes to zero, the model converges to a standard continuous-time model in which transitions are spread uniformly across time. In line with empirical evidence, firms' wage offers are assumed to have a take-it-or-leave-it character, customized to the current wage of the worker.

In this environment, I show that period length crucially affects various equilibrium outcomes, including monthly flows between employment states, the unemployment rate, unemployment durations, the average wage, the degree of wage dispersion, and the shape of the earnings density. Unlike past literature or the continuous-time version of my model, a positive period length generates a wage ladder which workers climb at stochastic speed and which generates a non-monotonic earnings density.

These findings have important implications for empirical research. For example, they imply that wrong assumptions about the period length may lead to incorrect estimates of matching or separation rates, productivity distributions, or other fundamental parameters of labor market models. Hence, the choice of a period length should be discussed and motivated explicitly. For this, as well as for building more realistic models of clustering, more precise data on the transitions that workers make between employment states will be essential.

[28] This extends Carrillo-Tudela (2009a) who obtains a similar result, but only for the transition between the first and the second job. After that, customization is no longer possible in his setup, which eliminates the inefficiency.

Appendix

Proof of Lemma 1

The proof is similar to the one of lemma 1 in Burdett and Judd (1983).[29] The matching technology implies that a worker gets at least two job offers with probability $1 - p_s(0) - p_s(1) > 0$. In this case, the worker will compare the offers and accept the best one. This feature implies that if all firms offer the same wage $r(w) < x < y$, a deviant can do better by offering a marginally higher wage, which allows it to attract workers that compare multiple job offers with probability 1. Offering $x = y$ however, leading to a match payoff of zero, is dominated by offering $x = r(w)$ since there is a strictly positive probability $p_s(1)$ that the candidate does not compare wages. Hence, firms offer wages according to a mixed strategy. Similar arguments rule out mass points and gaps in the support.

Proof of Lemma 2

First, note that the infimum of the support of $F(x|w)$ must equal $r(w)$. Offers weakly below $r(w)$ are rejected and give a payoff of zero. On the other hand, if the infimum of the support would be strictly larger than $r(w)$, the firm offering the lowest wage could decrease its offer and make a higher profit. Second, let $\bar{x}(w)$ denote the upper bound of the support of $F(x|w)$, hence $\mathcal{F}(w) = (r(w), \bar{x}(w)]$. The equilibrium definition then implies that the payoff for the firm must be the same for each $x \in \mathcal{F}(w)$. Hence, an expression for the wage offer distribution $F(x|w)$ for $x \in \mathcal{F}(w)$ follows from the equal profit condition $\lim_{x' \to r(w)} V_V(x'|w) = V_V(x|w)$, which is equivalent to

$$\frac{e^{-\lambda_s \tau} \rho (1-\phi)(y - r(w))}{1 - \rho p_E(0)(1-\phi)} = \frac{e^{-\lambda_s \tau (1 - F(x|w))} \rho (1-\phi)(y-x)}{1 - \rho p_E(0)(1-\phi)}.$$

Solving for $F(x|w)$ yields

$$F(x|w) = \frac{1}{\lambda_s \tau} \log\left(\frac{y - r(w)}{y - x}\right).$$

The presented supremum $\bar{x}(w)$ of the support of $F(x|w)$ follows from solving $F(\bar{x}(w)|w) = 1$.

Proof of Lemma 3

In order to derive the reservation wage, consider the Bellman equations (1) and (2). Both satisfy Blackwell's (1965) sufficient conditions for a contraction mapping, implying that a unique solution for V_U and $V_E(w)$ exists. The reservation wage property implies that $V_E(r(h)) = V_U$. Conjecture that the solution for $V_E(w)$ is a linear function of w, i.e., $V_E(w) = V_U + \gamma(w - r(h))$ for some unknown

[29] See also lemma 1 in Gautier and Moraga-Gonzalez (2004).

constant γ. Substituting this into (2) yields

$$V_E(w) = (1 + \rho\gamma(1-\phi)\, p_E(0))\, w + \rho V_U - \rho\gamma(1-\phi)\, r(h) + \rho\gamma(1-\phi) \sum_{j=1}^{\infty} p_E(j) \int_{w}^{\bar{x}(w)} x dF^j(x|w) \tag{14}$$

Note that for all $r(w) < x \leq \bar{x}(w)$

$$\begin{aligned} \sum_{j=1}^{\infty} p_s(j)\, F^j(x|w) &= e^{-\lambda_s \tau (1 - F(x|w))} - e^{-\lambda_s \tau}, \\ &= e^{-\lambda_s \tau} \frac{y - r(w)}{y - x} - e^{-\lambda_s \tau}. \end{aligned} \tag{15}$$

where the first equality follows from the fact that $p_s(j)$ is Poisson and the second from the equilibrium expression for $F(x|w)$ as given in (7). Consequently,

$$\begin{aligned} \sum_{j=1}^{\infty} p_E(j) \int_{r(w)}^{\bar{x}(w)} x dF^j(x|w) &= \int_{r(w)}^{\bar{x}(w)} x d \sum_{j=1}^{\infty} p_E(j)\, F^j(x|w) \\ &= e^{-\lambda_E \tau} \int_{r(w)}^{\bar{x}(w)} x d \frac{y - r(w)}{y - x}. \end{aligned}$$

Integration by parts gives

$$\begin{aligned} \int_{r(w)}^{\bar{x}(w)} x d \frac{y - r(w)}{y - x} &= \bar{x}(w) \frac{y - r(w)}{y - \bar{x}(w)} - r(w) + (y - r(w)) \left[\log(y - \bar{x}(w)) - \log(y - r(w)) \right] \\ &= \left(e^{\lambda_E \tau} - 1 \right) y - \lambda_E \tau (y - r(w)). \end{aligned}$$

Hence

$$\sum_{j=1}^{\infty} p_E(j) \int_{r(w)}^{\bar{x}(w)} x dF^j(x|w) = \lambda_E \tau e^{-\lambda_E \tau} r(w) + \left(1 - e^{-\lambda_E \tau} - \lambda_E \tau e^{-\lambda_E \tau} \right) y. \tag{16}$$

This result can be used to rewrite (14) as follows

$$\begin{aligned} V_E(w) &= w + \rho V_U + \rho\gamma(1-\phi) \left[(1 - \Phi_E)(w - r(h)) + \Phi_E (y - r(h)) \right] \qquad (17) \\ &= \left[r(h) + \rho V_U + \rho\gamma(1-\phi)\, \Phi_E (y - r(h)) \right] + \left[1 + \rho\gamma(1-\phi)(1 - \Phi_E) \right] (w - r(h)). \end{aligned}$$

where

$$\Phi_s = 1 - e^{-\lambda_s \tau} - \lambda_s \tau e^{-\lambda_s \tau}.$$

This confirms the linear structure of $V_E(w)$. In a similar way, one can obtain the following expression for V_U

$$
\begin{aligned}
V_U &= h+\rho V_U-\rho\gamma(1-\phi)(1-p_U(0))\,r(h)+\rho\gamma(1-\phi)\sum_{j=1}^{\infty}p_U(j)\int_{r(h)}^{\overline{x}(h)}x dF^j(x|h)\\
&= h+\rho V_U+\rho\gamma(1-\phi)\,\Phi_U(y-r(h)).
\end{aligned}
$$

The expressions for γ and the reservation wage $r(h)$ can be found by solving the system

$$
\begin{aligned}
(1-\rho)V_U &= r(h)+\rho\gamma(1-\phi)\,\Phi_E(y-r(h))\\
\gamma &= 1+\rho\gamma(1-\phi)(1-\Phi_E)\\
(1-\rho)V_U &= h+\rho\gamma(1-\phi)\,\Phi_U(y-r(h)).
\end{aligned}
$$

This yields

$$
\gamma=\frac{1}{1-\rho(1-\phi)(1-\Phi_E)}
$$

and

$$
r(h)=\frac{h+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)\,y}{1+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)}.
$$

Proof of Proposition 1

Lemmas 2 and 3 describe the unique equilibrium candidates for the wage offer distribution and the reservation wage function. In order to establish existence of this equilibrium, two things still need to be determined: 1) the existence of a stationary unemployment rate u, and 2) the absence of profitable deviations for firms or workers.

Consider the unemployment rate first. Steady state requires that inflow equals outflow. Note that workers who are unemployed at the beginning of a period are no longer unemployed one period later if they get at least one job offer and are not immediately hit by a job destruction shock. Hence, their matching probability is $\mu_U=\left(1-e^{-\lambda_U\tau}\right)(1-\phi)$, implying that outflow is equal to $\mu_U u$. On the other hand, a fraction ϕ of the employed workers loses its job such that inflow equals $\phi(1-u)$. Solving for u yields

$$
u=\frac{\phi}{\phi+\mu_U}.
$$

Next, I analyze whether there exist profitable deviations from the candidate equilibrium. Consider firms first. In the candidate equilibrium, a firm meeting a worker currently earning a wage $w\in(r(h),y)$ will randomly offer a wage $x\in(w,\overline{x}(w)]$ and obtain a payoff $V_V(x|w)>0$. A few alternative strategies exist. First, the firm can switch to a wage offer $x'\leq w$. Clearly, such a deviation is not profitable, since the worker will reject the offer, implying a zero payoff for the firm. Second, the firm can offer $x'\in(w,\overline{x}(w)]$, where $x'\neq x$. Since $F(x|w)$ is constructed from a equal-profit condition on $(w,\overline{x}(w)]$, this deviation has no effect on the firm's payoff. Finally, the firm can make an offer

$x' > \bar{x}(w)$. In that case, the firm obtains a strictly lower payoff, $V_V(x'|w) < V_V(\bar{x}(w)|w) = V_V(x|w)$. Hence, no profitable deviations exist for firms.

Finally, consider deviations by workers. In the candidate equilibrium, a worker accepts or rejects wage offers based on the reservation wage function characterized in (10). This yields unemployed workers a payoff V_U and workers who are employed at a wage w a payoff $V_E(w)$. An unemployed worker may then deviate by accepting a wage offer $x < r(h)$. This yields the worker a payoff $V_E(x)$, which by the monotonicity of V_E is strictly lower than $V_E(r(h)) = V_U$, and therefore not profitable. Likewise, rejecting a wage offer $x > r(h)$ implies a payoff $V_U = V_E(r(h)) < V_E(x)$. Profitable deviations for employed workers can be ruled out in a similar fashion; $V_E(x) \lesseqgtr V_E(w)$ for $x \lesseqgtr w$.

Proof of Lemma 4

Preliminaries

Recall that I define the following cutoff points

$$\{\hat{w}_0, \hat{w}_1, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} = \{r(h), \hat{w}_{E,1}, \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\} .$$

The earnings distribution on the interval $(\hat{w}_{n-1}, \hat{w}_n]$ is denoted by $G_n(w)$ and the corresponding density by $g_n(w)$. I then derive the earnings distribution in a recursive way. First, I obtain a closed-form expression for $G_1(w)$. Subsequently, I derive a recursive relationship between $G_n(w)$ and $G_{n-1}(w)$ for all $n \in \{2, \ldots, \hat{n}+1\}$. Finally, I show that knowledge of $G_{n-1}(w)$ and $G_{n-2}(w)$ is sufficient to derive $G_n(w)$ for all $n \in \{\hat{n}+2, \ldots\}$. In each case, I use the steady state nature of the equilibrium to determine the solution. That is, I consider $\mathcal{Z}(w)$, the set of workers earning less than w, and equate inflow to outflow to obtain the following condition, which can be turned into a differential equation.

$$\frac{u}{1-u}(1-\phi)\sum_{j=1}^{\infty} p_U(j) F^j(w|h) = \phi G(w) + (1-\phi)\int_{r(h)}^{w} \sum_{j=1}^{\infty} p_E(j)\left(1 - F^j(w|z)\right) g(z)\, dz. \tag{18}$$

Throughout the proof I simplify notation by letting Δ_E denote the average duration of a match (in time periods), i.e., $\Delta_E = \frac{1}{\phi+\mu_E}$.

Derivation of $G_1(w)$

I derive an expression for $G_1(w)$ by considering $\mathcal{Z}(w)$ for values of w in the interval $(r(h), \hat{w}_1]$. First, recall from equation (15) that

$$\sum_{j=1}^{\infty} p_s(j) F^j(w|z) = \begin{cases} 0 & w \leq r(z) \\ e^{-\lambda_s \tau}\left(\frac{y-r(z)}{y-w} - 1\right) & w \in (r(z), \bar{x}(z)] \\ 1 - e^{-\lambda_s \tau} & w > \bar{x}(z). \end{cases} \tag{19}$$

Substituting this into inflow (12) and outflow (13) and equating the resulting expressions yields the following condition

$$\frac{u}{1-u}\left(1-\phi-\mu_U\right)\left(w-r\left(h\right)\right)=\left(\phi+\mu_E\right)\left(y-w\right)G_1\left(w\right)-\left(1-\phi-\mu_E\right)\int_{r(h)}^{w}\left(w-z\right)g_1\left(z\right)dz.$$

Taking the first derivative with respect to w turns this expression into a differential equation. Simplifying the result by substituting $\frac{u}{1-u}=\frac{\phi}{\mu_U}$ yields

$$g_1\left(w\right)-\frac{\Delta_E}{y-w}G_1\left(w\right)=\frac{\phi}{\mu_U}\left(1-\phi-\mu_U\right)\frac{\Delta_E}{y-w}.$$

This is a first-order non-homogeneous linear differential equation. Hence, the solution for $G_1\left(w\right)$ is given by

$$G_1\left(w\right) \quad = \quad \exp\left(\Delta_E\int\frac{1}{y-w}dw\right)\left(\int\exp\left(-\Delta_E\int\frac{1}{y-w}dw\right)\frac{\phi}{\mu_U}\left(1-\phi-\mu_U\right)\frac{\Delta_E}{y-w}dw+C_1\right),$$

where C_1 is a constant. Solving the integrals and simplifying the resulting expression gives

$$G_1\left(w\right) \quad = \quad C_1\left(y-w\right)^{-\Delta_E}-\frac{\phi}{\mu_U}\left(1-\phi-\mu_U\right),$$

The value of the constant C_1 follows from the condition $G_1\left(r\left(h\right)\right)=0$. This implies

$$C_1=\frac{\phi}{\mu_U}\left(1-\phi-\mu_U\right)\left(y-r\left(h\right)\right)^{\Delta_E}.$$

Hence, we obtain the following closed-form solution for the earnings density on $\left(r\left(h\right),\hat{w}_1\right]$

$$G_1\left(w\right)=\frac{\phi}{\mu_U}\left(1-\phi-\mu_U\right)\left(\left(\frac{y-r\left(h\right)}{y-w}\right)^{\Delta_E}-1\right). \tag{20}$$

Derivation of $G_2\left(w\right),\ldots,G_{\hat{n}+1}\left(w\right)$

Next, consider $\mathcal{Z}\left(w\right)$ for $w\in\left(\hat{w}_{n-1},\hat{w}_n\right]$, $n\in\{2,3,\ldots\hat{n}+1\}$. In these intervals, inflow still depends on w, as in the derivation of $g_1\left(w\right)$. However, not all workers can leave $\mathcal{Z}\left(w\right)$ anymore by a job-to-job transition. The workers earning low wages will stay in $\mathcal{Z}\left(w\right)$ even if they experience the largest possible wage increase when moving to a new job. Let $\underline{w}\left(w\right)$ denote the minimum wage that a worker must earn in order to be able to earn $w>\hat{w}_1$ in his next job. By inverting equation (8), I obtain

$$\underline{w}\left(w\right)=e^{\lambda_E\tau}w+\left(1-e^{\lambda_E\tau}\right)y. \tag{21}$$

Note that $\underline{w}(w)$ is strictly increasing in w and that $\underline{w}(\hat{w}_{n-1}) = \hat{w}_{n-2}$. Hence, $F(w|z) = 1$ for $z < \underline{w}(w)$, such that the integral on the right hand side of (18) can start at $\underline{w}(w)$. Substituting the relevant cases of equation (19) then gives

$$\frac{u}{1-u}(1-\phi-\mu_U)\left(\frac{w-r(h)}{y-w}\right) = G(w) - (1-\phi)G(\underline{w}(w)) - (1-\phi-\mu_E)\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz. \tag{22}$$

Note that $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-1}(\underline{w}(w))$, and that $\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz$ can be rewritten as

$$\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz = \int_{\underline{w}(w)}^{\hat{w}_{n-1}}\frac{y-z}{y-w}g_{n-1}(z)\,dz + \int_{\hat{w}_{n-1}}^{w}\frac{y-z}{y-w}g_n(z)\,dz.$$

Again, a differential equation can be created by multiplying both the left hand side and the right hand side with $y-w$ and taking the first derivative with respect to w. For the left hand side of (22) this results in $\frac{\phi}{\mu_U}(1-\phi-\mu_U)$. For the right hand side we get

$$\frac{d}{dw}G_n(w)(y-w) = g_n(w)(y-w) - G_n(w),$$

$$\frac{d}{dw}G_{n-1}(\underline{w}(w))(y-w) = g_{n-1}(\underline{w}(w))\,e^{\lambda_E\tau}(y-w) - G_{n-1}(\underline{w}(w)),$$

and

$$\begin{aligned}\frac{d}{dw}\int_{\underline{w}(w)}^{w}(y-z)\,g(z)\,dz &= (y-w)\,g_n(w) - e^{\lambda_E\tau}(y-\underline{w}(w))\,g_{n-1}(\underline{w}(w)).\\ &= (y-w)\,g_n(w) - e^{2\lambda_E\tau}(y-w)\,g_{n-1}(\underline{w}(w)).\end{aligned}$$

Hence, we get the following first-order non-homogeneous linear differential equation

$$g_n(w) - \frac{\Delta_E}{y-w}G_n(w) = \frac{\Delta_E}{y-w}\left(\frac{\phi}{\mu_U}(1-\phi-\mu_U) - (1-\phi)\,G_{n-1}(\underline{w}(w))\right).$$

The solution is equal to

$$G_n(w) = C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) - (1-\phi)\,\Delta_E\,(y-w)^{-\Delta_E}\int(y-w)^{\Delta_E-1}G_{n-1}(\underline{w}(w))\,dw.$$

where the constant C_n follows from the condition $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$. Finally, integration by parts gives

$$G_n(w) = C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) + (1-\phi)\left(G_{n-1}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int(y-w)^{\Delta_E}\,dG_{n-1}(\underline{w}(w))\right).$$

Derivation of $G_{\hat{n}+2},\ldots$

Next, consider $\mathcal{Z}(w)$ for $w \in (\hat{w}_{n-1}, \hat{w}_n]$, $n \in \{\hat{n}+2, \hat{n}+3, \ldots\}$. In these intervals the inflow no longer depends on w, since unemployed workers finding a job are always hired at a wage below $\hat{w}_n$. Hence, inflow equals

$$I(w) = u\mu_U.$$

Outflow still depends on w, but again the integral in the right hand side of (18) starts at $\underline{w}(w)$. Hence, the earnings distribution is implied by

$$\frac{u}{1-u}\mu_U = G(w) - (1-\phi)\,G(\underline{w}(w)) - (1-\phi-\mu_E)\int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\,dz. \tag{23}$$

Note that now both $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ contribute to the cutoff points, implying that there are twice as many intervals as for $w < \hat{w}_{U,1}$. More specifically, there are two cutoff points between any w and $\underline{w}(w)$. Hence, $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-2}(\underline{w}(w))$, and $\int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\,dz$ can be rewritten as

$$\int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\,dz = \int_{\underline{w}(w)}^{\hat{w}_{n-2}} \frac{y-z}{y-w} g_{n-2}(z)\,dz + \int_{\hat{w}_{n-2}}^{\hat{w}_{n-1}} \frac{y-z}{y-w} g_{n-1}(z)\,dz + \int_{\hat{w}_{n-1}}^{w} \frac{y-z}{y-w} g_n(z)\,dz.$$

Create again a differential equation by multiplying both the left hand side and the right hand side of (23) with $y-w$ and taking the first derivative with respect to w. For the left hand side, this yields $-\frac{u}{1-u}e^{-\delta}\left(1-e^{-\lambda_U}\right) = -\phi$. On the right hand side we get

$$\frac{d}{dw}G_n(w)(y-w) = g_n(w)(y-w) - G_n(w),$$

$$\frac{d}{dw}G_{n-2}(\underline{w}(w))(y-w) = g_{n-2}(\underline{w}(w))\,e^{\lambda_E\tau}(y-w) - G_{n-2}(\underline{w}(w)),$$

and

$$\begin{aligned}\frac{d}{dw}\int_{\underline{w}(w)}^{w}(y-z)\,g(z)\,dz &= (y-w)\,g_n(w) - e^{\lambda_E\tau}(y-\underline{w}(w))\,g_{n-2}(\underline{w}(w)).\\ &= (y-w)\,g_n(w) - e^{2\lambda_E\tau}(y-w)\,g_{n-2}(\underline{w}(w)).\end{aligned}$$

Hence, we obtain the following first-order non-homogeneous linear differential equation.

$$g_n(w) - \frac{\Delta_E}{y-w}G_n(w) = -\frac{\Delta_E}{y-w}\left(\phi + (1-\phi)\,G_{n-2}(\underline{w}(w))\right)$$

The solution is equal to

$$G_n(w) = C_n + \phi - (1-\phi)\,\Delta_E\,(y-w)^{-\Delta_E}\int (y-w)^{\Delta_E-1}\,G_{n-2}(\underline{w}(w))\,dw,$$

where the constant C_n follows from the condition $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$. Finally, integration by parts gives

$$G_n(w) = C_n + \phi - (1-\phi)\left(G_{n-2}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int (y-w)^{\Delta_E - 1}\, dG_{n-2}(\underline{w}(w))\right). \tag{24}$$

Proof of Lemma 5

Note that the second derivative of $G_1(w)$ equals

$$g_1'(w) \quad = \quad \frac{\phi}{\mu_U}(1-\phi-\mu_U)\Delta_E(\Delta_E+1)(y-r(h))^{\Delta_E}(y-w)^{-\Delta_E-2}.$$

This is positive for all $w \in (r(h), \hat{w}_1]$, implying that the first statement holds.

Further, earning a wage $w > \hat{w}_{U,n}$ requires at least n consecutive job-to-job transitions without experiencing a job destruction shock. The conditional probability that a match ends with a job-to-job transition is $\frac{\mu_E}{\phi+\mu_E}$. Hence, the probability of experiencing n job-to-job transitions in a row is $\left(\frac{\mu_E}{\phi+\mu_E}\right)^n$ which tends to 0 if $n \to \infty$. Hence, $g(w) \downarrow 0$ if $w \to y$.

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Figure 1: Cutoff Points

Cutoff points of the earnings distribution. Note that $\hat{w}_{E,n} < \hat{w}_{U,1}$ for $n \leq \hat{n}$ (here $\hat{n} = 2$). After $\hat{w}_{E,\hat{n}}$, the cutoff points formed by $\hat{w}_{U,i}$ and $\hat{w}_{E,\hat{n}+i}$ alternate.

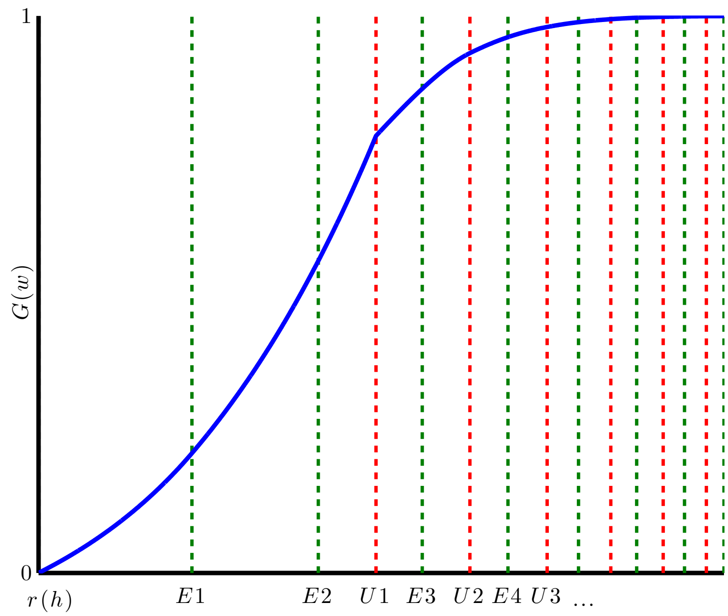

Table 1: End Dates of Unemployment Spells – Regression Results

IRR = incidence-rate ratio relative to the base category (1 for dates; Monday for days); s.e. = robust standard errors. Regression statistics: estimated constant = 79,210 (18,684); $R^2 = 0.94$; based on 217 day / date combinations and 691,844 spells.

Date	IRR	s.e.	Date	IRR	s.e.	Date	IRR	s.e.	Day	IRR	s.e.
1	1.00	–	**11**	0.05	0.01	**21**	0.05	0.01	**Mon**	1.00	–
2	0.11	0.03	**12**	0.05	0.01	**22**	0.05	0.01	**Tue**	0.48	0.04
3	0.07	0.02	**13**	0.05	0.01	**23**	0.05	0.01	**Wed**	0.38	0.03
4	0.06	0.01	**14**	0.05	0.01	**24**	0.05	0.01	**Thu**	0.37	0.03
5	0.06	0.01	**15**	0.09	0.02	**25**	0.05	0.01	**Fri**	0.30	0.02
6	0.06	0.01	**16**	0.06	0.01	**26**	0.04	0.01	**Sat**	0.19	0.02
7	0.05	0.01	**17**	0.05	0.01	**27**	0.04	0.01	**Sun**	0.12	0.01
8	0.05	0.01	**18**	0.05	0.01	**28**	0.08	0.02			
9	0.05	0.01	**19**	0.05	0.01	**29**	0.06	0.02			
10	0.06	0.01	**20**	0.06	0.01	**30**	0.27	0.08			
						31	0.40	0.13			

Table 2: Begin Dates of Unemployment Spells – Regression Results

IRR = incidence-rate ratio relative to the base category (1 for dates; Monday for days); s.e. = robust standard errors. Regression statistics: estimated constant = 22,531 (5,735); $R^2 = 0.97$; based on 217 day / date combinations and 691,844 spells.

Date	IRR	s.e.	**Date**	IRR	s.e.	**Date**	IRR	s.e.	**Day**	IRR	s.e.
1	1.00	–	**11**	0.21	0.05	**21**	0.21	0.05	**Mon**	1.00	–
2	0.31	0.08	**12**	0.21	0.05	**22**	0.20	0.05	**Tue**	0.84	0.03
3	0.29	0.08	**13**	0.21	0.05	**23**	0.20	0.05	**Wed**	0.66	0.02
4	0.27	0.07	**14**	0.20	0.05	**24**	0.19	0.05	**Thu**	0.66	0.02
5	0.26	0.07	**15**	0.24	0.06	**25**	0.18	0.05	**Fri**	0.56	0.02
6	0.25	0.07	**16**	0.26	0.07	**26**	0.19	0.05	**Sat**	0.24	0.01
7	0.24	0.06	**17**	0.21	0.05	**27**	0.20	0.05	**Sun**	0.08	0.01
8	0.22	0.06	**18**	0.21	0.05	**28**	0.22	0.06			
9	0.22	0.06	**19**	0.21	0.05	**29**	0.21	0.06			
10	0.23	0.06	**20**	0.21	0.05	**30**	0.25	0.06			
						31	0.17	0.05			

Table 3: Effect of a Change in the Period Length

Equilibrium outcomes for various values of the period length τ (bold values: baseline calibration). The first table reports the periodical transition probabilities ($\mu_U(\tau)$, $\mu_E(\tau)$, and $\phi(\tau)$), the monthly flows between unemployment and employment (UE) and vice versa (EU), the unemployment rate ($u(\tau)$), and the expected unemployment duration $(D(\tau))$. The monthly flows are omitted for period lengths longer than 1 month. The second table presents some statistics regarding the earnings distribution (relative to y), with the Mm-ratio referring to the mean-min ratio.

τ	μ_U	μ_E	ϕ	UE	EU	u	D
0.00	0.000	0.000	0.000	0.449	0.018	0.039	1.65
0.12	0.073	0.004	0.003	0.448	0.019	0.040	1.72
0.25	0.140	0.008	0.006	0.448	0.019	0.042	1.79
0.50	0.258	0.016	0.012	0.446	0.021	0.045	1.94
1.00	**0.443**	**0.032**	**0.024**	**0.443**	**0.024**	**0.051**	**2.26**
2.00	0.668	0.061	0.047	–	–	0.066	2.99
4.00	0.827	0.113	0.093	–	–	0.101	4.84
6.00	0.841	0.157	0.136	–	–	0.139	7.13
12.00	0.747	0.246	0.253	–	–	0.253	16.07

τ	Min	Mean	Std. dev.	Mm-ratio
0.00	0.400	0.401	0.000	1.001
0.12	0.659	0.672	0.007	1.021
0.25	0.754	0.773	0.010	1.026
0.50	0.832	0.858	0.013	1.031
1.00	**0.884**	**0.918**	**0.015**	**1.038**
2.00	0.909	0.955	0.016	1.051
4.00	0.905	0.979	0.016	1.082
6.00	0.885	0.990	0.014	1.118
12.00	0.799	0.999	0.005	1.250

Table 4: Effect of a Change in the Job Offer Arrival Rates

Equilibrium outcomes for various values of the job offer arrival rates λ_U and λ_E (bold values: baseline calibration). The first table reports the periodical transition probabilities ($\mu_U(\tau)$, $\mu_E(\tau)$, and $\phi(\tau)$), the monthly flows between unemployment and employment (UE) and vice versa (EU), the unemployment rate ($u(\tau)$), and the expected unemployment duration $(D(\tau))$. The second table presents some statistics regarding the earnings distribution (relative to y), with the Mm-ratio referring to the mean-min ratio.

λ_U	λ_E	μ_U	μ_E	ϕ	UE	EU	u	D
0.00	0.00	0.000	0.000	0.024	0.000	0.024	1.000	∞
0.08	0.00	0.073	0.004	0.024	0.073	0.024	0.252	13.77
0.15	0.01	0.140	0.008	0.024	0.140	0.024	0.149	7.17
0.30	0.02	0.258	0.016	0.024	0.258	0.024	0.086	3.88
0.60	**0.03**	**0.443**	**0.032**	**0.024**	**0.443**	**0.024**	**0.051**	**2.26**
1.21	0.07	0.668	0.061	0.024	0.668	0.024	0.034	1.50
2.42	0.13	0.827	0.113	0.024	0.827	0.024	0.026	1.21
3.63	0.20	0.841	0.157	0.024	0.841	0.024	0.025	1.19
7.26	0.40	0.747	0.246	0.024	0.747	0.024	0.024	1.34

λ_U	λ_E	Min	Mean	Std. dev.	Mm-ratio
0.00	0.00	0.400	0.400	0.000	1.001
0.08	0.00	0.451	0.472	0.012	1.046
0.15	0.01	0.558	0.591	0.018	1.060
0.30	0.02	0.738	0.777	0.020	1.053
0.60	**0.03**	**0.884**	**0.918**	**0.015**	**1.038**
1.21	0.07	0.950	0.976	0.009	1.028
2.42	0.13	0.969	0.995	0.005	1.026
3.63	0.20	0.969	0.998	0.003	1.030
7.26	0.40	0.947	1.000	0.001	1.056