Applications and Interviews

Firms' Recruiting Decisions in a Frictional Labor Market*

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Abstract

I develop a directed search model to study the recruitment decisions of firms competing for workers who ex post differ in two dimensions: i) their match productivity, and ii) their probability of accepting a job offer, endogenously determined by their choice of application portfolio. To attract these workers, firms post a *recruiting intensity* and a *hiring standard*, in addition to terms of trade. A higher recruiting intensity is costly, but allows the firm to select more applicants for an interview, which reveals their productivity. The hiring standard solves the tradeoff between immediate hiring and waiting for a potentially better match in the future. I characterize equilibrium and find that various outcomes, including uniqueness of equilibrium and the cyclicality of recruiting intensity, crucially depend on firms' recruiting cost and workers' search cost. Calibration of the model to the US labor market indicates a continuum of equilibria. Given selection of a particular equilibrium, hiring standards are countercyclical while recruiting intensity is procyclical. The calibrated model creates more amplification than a standard model without intensive margins and gives rise to procyclical match efficiency when viewed through the lens of a Cobb-Douglas matching function.

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1 Introduction

1.1 Motivation and Summary

Google, a company with less than 60,000 employees, annually receives more than 3 million applications from job seekers. Through an elaborate series of assessments and interviews with recruiters and potential colleagues, this pool gets narrowed down further and further until eventually roughly 0.25% is hired.[1] Although the numbers vary, the recruiting process at other firms similarly involves numerous decisions and substantial costs; for firms covered by the National Employer Survey 1997, Villena-Roldan (2012) documents a median of 5 interviews per vacancy and an average labor cost of screening applicants of 4200 dollars per hire.[2]

Search models of the labor market based on the Diamond-Mortensen-Pissarides (DMP) framework have traditionally abstracted from these aspects of the matching process. Instead, these models assume that the entire process can be summarized by an aggregate matching function which solely depends on the ratio of the number of vacancies to the number of unemployed. Although this approach has been very fruitful for improving our understanding of modern labor markets, recent quantitative work has pointed out limitations and drawn attention to firms' recruiting decisions while doing so.

For example, in response to Shimer (2005b) arguing that DMP models cannot explain the empirical volatility of unemployment and vacancies over the business cycle, Hagedorn and Manovskii (2008) reconcile data and theory by reducing the size of the match surplus in the calibration, justifying this with evidence from the 1982 Employment Opportunities Pilot Projects (EOPP) data set that firms do not spend much time on screening applicants. Further, Davis et al. (2013) have pointed out that the efficiency of the matching process—as measured by the number of matches for a given number of vacancies and unemployed—is not time-invariant but rather procyclical, viewed through the lens of a standard Cobb-Douglas matching function. They attribute this to firms' recruiting decisions; as they explain, focusing exclusively on the number of vacancies ignores the fact that firms may increase their job-filling probability if they "increase advertising or search intensity per vacancy, screen applicants more quickly, relax hiring standards, improve working conditions, and offer more attractive compensation to prospective employees."

The strategic use of wages plays a central role in models of directed search, pioneered by Moen (1997), but decisions regarding screening and hiring standards are still relatively unexplored in

[1] For details on Google's hiring process, see Bock (2014) and Google (2016).

[2] The Employment Opportunities Pilot Projects data set, which I discuss in detail in section 4, indicates that US firms recruiting for low-skilled jobs in the early eighties on average received 9.7 applications, conducted 5.7 interviews and made 1.1 job offers per hire, spending 11 hours of managerial time on the process. Marinescu and Wolthoff (2015) document an average of 59 applications per job ad using CareerBuilder data from 2011. Burks et al. (2015) report that an undisclosed high-tech firm with 1.4 million applicants between 2003 and 2011 subjected 10% to a series of interviews and made a job offer to 0.5%. Weber (2012) reports that in 2011 Starbucks and Proctor & Gamble both hired less than 1% of their 7.6 million and nearly 1 million applicants, respectively. She also describes how the vast majority of the 500 largest US firms has started to use specialized software to automate the screening of résumés in an attempt to reduce recruiting costs.

the search literature.[3] As a result, various important questions regarding these decisions remain unanswered. For example, what are the tradeoffs that firms face in their recruiting process? If we observe that firms do not spend much time on screening, as in the EOPP data, is that because they screen most applicants at low cost, or because they screen few applicants at high cost? What are the implications of either scenario for labor market outcomes? Finally, how do firms' recruiting decisions respond to changes in aggregate conditions, such as a productivity shock?

This paper aims to shed light on these questions by presenting a micro-founded directed search model of the labor market which includes meaningful recruiting decisions for firms. That is, in addition to selecting workers' compensation, each firm makes two other choices: a *recruiting intensity*, determining how many applicants the firm will interview, and a *hiring standard*, describing which applicants the firm will or will not consider hiring.[4] The primary reason for firms to interview applicants is an information friction: applicants differ in their productivity and a firm can learn this productivity only through a job interview. A larger number of interviews allows a firm to rank more applicants and therefore hire better workers, but comes at a higher cost, capturing the idea that each interview requires time or effort. The hiring standard deals with a similar tradeoff in an intertemporal context; by being more selective about whom to hire, a firm can ensure that it will form better matches, but at the cost of remaining unmatched for longer.

Recruitment is not only a matter of identifying a productive applicant: the candidate also needs to be willing to accept the job offer. The EOPP data indicates that rejection of job offers happens quite frequently (up to 10%) and surveys among recruiters suggest that the cause often lies in the simultaneous arrival of a financially more attractive offer from a different firm (CareerBuilder, 2014; Management Recruiters International, 2016). To capture this dimension of the recruiting process, I allow workers to send multiple applications per period, as in Galenianos and Kircher (2009) and Kircher (2009). Having an arbitrary number of both applications and interviews results in a complicated network of many-to-many relations between workers and firms. One contribution of this paper is to present a set of modeling choices which keeps the analysis of such an environment tractable, while unifying the existing literature.

I characterize equilibrium in this environment, derive a rich set of empirical predictions and address the above questions. I find that the answers crucially depend on workers' application behavior and firms' recruiting cost. To be precise, in the special case in which workers send one application per period, as is the standard assumption in the literature, equilibrium is always unique and firms' recruiting intensity is always countercyclical. In contrast, in the full model with workers sending multiple applications per period, uniqueness of equilibrium only arises when firms' recruiting cost is sufficiently small, while a continuum of equilibria exists when the recruiting cost is sufficiently large. Moreover, firms' recruiting intensity may now be procyclical.

The complexity of the equilibrium with multiple applications prevents a precise analytical char-

[3] See section 1.2 for a literature review.

[4] Hence, unlike Davis et al. (2013), I use the terminology "recruiting intensity" in a very specific way (describing firms' interviewing decisions), and treat hiring standards and workers' compensation as separate choices. I maintain the standard assumption of one vacancy per firm, because firms' choice of how many vacancies to create has already been analyzed in detail by Kaas and Kircher (2015).

acterization of various comparative statics. In order to make progress, I therefore calibrate the model using the EOPP data. I find that although firms spend little time on recruiting, they learn the productivity of most applicants, as interviewing is not very costly. At the same time however, the recruiting cost is large enough to give rise to a continuum of equilibria. I consider two different equilibrium selection rules and show that firms' recruiting intensity is indeed procyclical. Together with the countercyclicality of their hiring standards, this yields a matching process which not only appears to be more efficient in booms when viewed through the lens of a Cobb-Douglas matching function, but also creates twice as much amplification in response to productivity shock than a standard search model.

Having provided a rough outline, I now describe the environment and the main results in more detail. As I discuss in section 2, I consider a directed search model of the labor market with a continuum of workers and free entry of firms, each with one vacancy. Workers choose a search intensity, determining the number of applications that they can send, while firms choose a recruiting intensity and a hiring standard, in addition to their terms of trade. Firms have the ability to post information about these choices, helping workers to direct their applications to the firms that offer the highest expected payoff. However, frictions arise because search and recruiting are costly and because agents cannot coordinate their actions. As a result, the number of applications that a firm receives is a random variable with a distribution that depends on the firm's announcement. Subsequently, each firm interviews a number of applicants, consistent with its choice of recruiting intensity. It learns their productivity and ranks them by their profitability in order to make job offers, subject to the hiring standard.

To develop intuition for some of the tradeoffs, the first part of section 3 analyzes the special case in which each worker sends one application per period, as standard in the literature on directed search. I show that a unique equilibrium exists in this environment.[5] In this equilibrium, all firms choose the same terms of trade, hiring standard and recruiting intensity. Firms will generically interview multiple but not all applicants, balancing the cost of an extra interview with the gain of potentially finding a better match. Each firm will make a job offer to its most productive interviewee, as long as his productivity exceeds the hiring standard, and—because each worker has only one chance to match—this job offer is always accepted.

I find that firms' hiring standards in this case are countercyclical: when aggregate productivity is high, it becomes more attractive to match quickly rather than wait, so firms become less selective and accept worse matches. The effect on firms' recruiting intensity is more complex, because there are two opposing effects. On the one hand, given a number of applicants, the productivity shock increases firms' payoff of a match, which causes firms to choose a higher recruiting intensity. On the other hand however, the productivity shock encourages firm entry, reducing the number of applicants at each firm. This causes firms to choose a lower recruiting intensity, as the number of applicants and the number of interviews are complements in hiring.[6] I show that the latter effect dominates, making recruiting intensity countercyclical.

[5]I prove in the online appendix that this equilibrium is constrained efficient.

[6]The number of interviews is necessarily bounded by the number of applicants.

While the assumption of a single application per worker is pervasive and analytically convenient, it is restrictive for a number of reasons. First, the assumption is at odds with with recent empirical evidence on workers' search behavior. For example, Belot et al. (2015) find that job seekers participating in their experiment send as much as 11 applications per week. Similarly, 2012 data from CareerBuilder.com, the largest US employment website, indicates that workers send on average 5.0 applications on days on which they apply at least once.[7] Second, a single application per worker implies a unique equilibrium contract, which prevents an interpretation of the large cross-sectional variation in recruitment decisions and outcomes in real life, documented by e.g. Davis et al. (2013). Finally, and perhaps most importantly, assuming a single application per worker affects the marginal benefit of an extra interview or a more selective hiring standard by ruling out rejection of job offers.

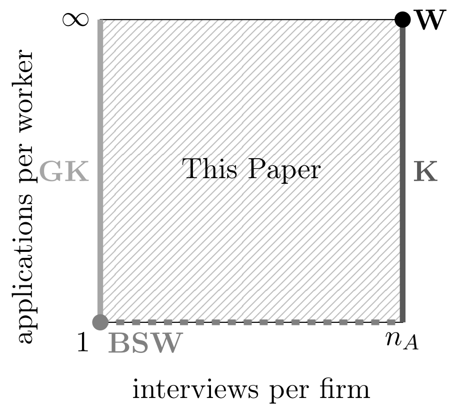
In the second part of section 3, I therefore return to the full model with multiple applications. As in Galenianos and Kircher (2009) and Kircher (2009), wage dispersion arises in this case. Hence, a worker choosing a higher search intensity incurs a higher cost, but reduces his risk of not getting any job offer and—conditional on getting an offer—increases his chances of finding a job with a high payoff. For firms, the introduction of rejection of job offers changes their tradeoffs in non-trivial ways. In equilibrium, firms posting different terms of trade choose different recruiting intensities, but the relationship is not necessarily monotonic, unlike the hiring standards which are increasing in the terms of trade. Interestingly, when their recruiting cost is sufficiently large, firms' choices are such that workers sending less than the maximum number of applications are indifferent between multiple different application portfolios, leading to a continuum of equilibria. Only when firms' recruiting cost is small enough, this multiplicity disappears and a unique equilibrium arises.

In section 4, I discuss how the model can be calibrated using aggregate matching and recruiting information from the Current Population Survey (CPS) and the EOPP data set. I show that while the model with one application per worker cannot match the data, the model with multiple applications does a good job and yields reasonable empirical predictions. Firms' recruiting cost is calibrated at 0.7% of monthly output, which is small enough for firms to learn the productivity of most applicants, but large enough to give rise to a continuum of equilibria. Focussing on the two equilibrium selection rules at opposite sides of the spectrum, I find that recruiting intensity is now procyclical, while hiring standards remain countercyclical. I demonstrate how this causes researchers analyzing the data created by my model using a Cobb-Douglas matching function to conclude that match efficiency is procyclical, in line with recent empirical findings. Finally, I analyze the volatility of labor market tightness—i.e. the ratio of vacancies to unemployment—in response to a productivity shock and find that my model creates twice as much amplification as a standard model.

The appendix contains omitted definitions and proofs. Additional details on the data and the calibration procedure as well as results on alternative contract structures and efficiency are relegated to the online appendix.

[7] I thank Ioana Marinescu and Roland Rathelot for this information. A detailed description of the data can be found in Marinescu and Rathelot (2014). Kudlyak et al. (2012) and Faberman and Kudlyak (2014) provide similar evidence of simultaneous search with data from SnagAJob.com, which specializes in hourly jobs.

Figure 1: Related Literature.



BSW = Burdett et al. (2001); GK = Galenianos and Kircher (2009); K = Kircher (2009); W = Walrasian outcome. n_A = firms' (Poisson-distributed) number of applicants. For illustrative purposes, the figure assumes that workers are homogeneous in productivity. Burdett et al. (2001) refer to workers and firms as "buyers" and "sellers", respectively; they assume that each firm can contact one worker, but relaxing this assumption does not change the equilibrium, as indicated by the dashed line.

1.2 Related Literature

This paper contributes to multiple strands of literature. From a theoretical point of view, this paper adds to work on search models of the labor market. The papers in this literature have explored various assumptions regarding the process that governs meetings between workers and firms. For example, DMP models, Moen (1997) and Acemoglu and Shimer (1999) consider bilateral meetings; Burdett et al. (2001), Shi (2001, 2002, 2006) and Shimer (2005a) use an urn-ball meeting technology, i.e. each worker applies to one firm and each firm faces a Poisson number of applicants; finally, Albrecht et al. (2006), Galenianos and Kircher (2009) and Kircher (2009) study extensions of the urn-ball framework by allowing workers to apply to multiple firms. However, in all of these specifications, the number of workers that a firm can contact and learn something about is exogenously determined and equal to either 1 or the number of applicants.[8] This paper contributes by making the number of interviews an endogenous choice. Figure 1 illustrates the connection with the most directly related papers.[9]

The only other equilibrium model with an endogenous number of interviews appears to be Villena-Roldan (2012), who studies how interview costs affect inequality in a random search model.

[8]Note that this distinction does not coincide with the distinction between bilateral and urn-ball, since some urn-ball models require firms to randomly select one applicant (e.g. Shi, 2001; Albrecht et al., 2006; Galenianos and Kircher, 2009; Gautier and Wolthoff, 2009). Recent work by Eeckhout and Kircher (2010), Lester et al. (2015, 2017) and Cai et al. (2016, 2017) has explored richer meeting processes in goods markets, but maintains—with exception of Lester et al. (2017)—the assumption that these processes are exogenous.

[9]The fact that meeting 1 or all applicants are feasible choices for the firms in my model does not necessarily imply that the papers displayed in the figure or cited above are nested. These papers generally differ along other dimensions and generate many insights that cannot be obtained from my model.

Relative to his work, I emphasize different questions and present a model which features search intensity decisions for workers and active competition among firms. Hiring standards have been considered more frequently, but typically only in environments with bilateral meetings between workers and firms (see e.g. Menzio and Shi, 2011; Moen and Rosén, 2011). A few papers introduce an intensive margin to firms' strategy in a different manner. For example, Pissarides (2000) analyzes recruiting intensity in a reduced-form way in a DMP model and finds that the equilibrium intensity is independent of any aggregate variable. Kaas and Kircher (2015) present a model with large firms and interpret recruiting intensity as the decision how many vacancies to create, which they show to be procyclical.

From an empirical point of view, this paper adds to the literature on recruiting decisions by firms. Most papers in this literature are primarily empirical and use partial equilibrium models; Barron et al. (1985), Barron et al. (1987) and Burdett and Cunningham (1998) study the determinants of firms' recruiting decisions with the EOPP data, while van Ours and Ridder (1992, 1993) and Abbring and van Ours (1994) analyze Dutch recruiting data and find evidence for simultaneous rather than sequential search by firms. I contribute by calibrating an equilibrium model, which makes it possible to study counterfactuals.

Finally, this paper is related to recent work on the ability of search models to capture business cycle fluctuations. Shimer (2005b) focusses on the volatility of labor market tightness, while work by Davis et al. (2013), Barnichon and Figura (2015) and Sedláček (2014, 2016) concerns the procyclicality of match efficiency. As I discuss in section 4, my model creates insights regarding both phenomena.

2 Model

2.1 Environment

Agents. Consider the steady state in a discrete-time labor market, populated by a mass 1 of workers and an endogenous measure of firms. Both types of agents are risk-neutral and maximize the expected sum of periodical payoffs, discounted at a factor $\beta \in (0, 1)$. All workers supply one indivisible unit of labor per period, while each firm has a position that can be filled by exactly one worker. All workers and all firms are ex ante homogeneous, but potential matches can differ in their productivity x, as I will describe below. As standard in the directed search literature, agents are restricted to symmetric and anonymous strategies.

Timing. Every period, the interaction between workers and firms takes place in a number of subsequent phases, in each of which agents' actions occur simultaneously. First, firms create vacancies and post contracts during the entry phase. After observing all posted contracts, unmatched workers choose their search intensity and send job applications during the search phase. Upon receiving these applications, firms interview a number of applicants during the recruitment phase. Matches are formed during the matching phase, after which output is produced in the production phase. In the last phase, job destruction takes place. I will now describe each of these phases in more detail.

Entry. In the entry phase, firms can create a vacancy at cost $k_V > 0$. Let v denote the measure of vacancies being created. Each entering firm makes a number of irreversible decisions regarding its recruitment strategy and the division of match output with the worker it will hire. For much of the analysis, it will be convenient to represent these choices by a triple, consisting of a firm's *hiring standard*, its *recruiting intensity*, and its *terms of trade*.

Hiring Standard. A firm's hiring standard $\underline{x} \in \mathcal{X} \equiv (0, \infty)$ is a threshold productivity level: by setting this standard, the firm commits to form the most productive match it identifies if the productivity x of this match exceeds $\underline{x}$ and to remain unmatched otherwise.

Recruiting Intensity. A firm's recruiting intensity $r \in \mathcal{R} \equiv [0, \infty)$ can be interpreted as a measure of the amount of time or resources that it will spend on interviewing workers. A higher value of r increases the probability that the firm will identify a very productive match, as I will explain below, but is also more costly.[10] The cost of a recruiting intensity r equals $k_R r$, where $k_R \in [0, \infty)$.

Terms of Trade. A firm's terms of trade $w \in \mathcal{W} \equiv (0, \infty)$ represent the expected value of employment that it promises to provide the worker it hires. For ease of exposition, I will initially remain agnostic about firms' precise implementation of w. After characterizing equilibrium, I will discuss in section 3.3 how firms can provide w in a way that satisfies individual rationality using a menu which conditions payments on the match productivity x.

Contracts. To reduce notation, I will often refer to a combination of $\underline{x}$, r and w as—for lack of a better term—a *contract*, denoted by $c = (\underline{x}, r, w) \in \mathcal{C} \equiv \mathcal{X} \times \mathcal{R} \times \mathcal{W}$. All identical contracts are treated symmetrically by workers and are therefore said to form a *submarket*.

Search. At the beginning of each period, an endogenous measure s of the workers is unmatched. These workers can apply to vacancies during the search phase. I assume the following structure. Unmatched workers first observe all posted contracts, allowing them to direct their search. Subsequently, they choose a search intensity $\alpha \in [0, \infty)$ at cost $k_A \alpha$, where $k_A \in [0, \infty)$. This search intensity can be interpreted as a measure of the time allocated to job search and it determines the number of applications that a worker can send. However, the relation between search intensity α and the number of applications a can be stochastic, e.g. because of randomness in the time it takes to apply to a particular vacancy.[11]

[10]The assumption that a firm chooses r before learning its number of applicants or their types captures the idea that the firm needs to hire a number of recruiters (or sign a contract with a recruiting agency) before knowing how successful its search for a worker is going to be.

[11]A key advantage of this approach is that it avoids optimization over a discrete variable. See Kaas (2010) for a related setup in a random search model.

Specifically, given a search effort α, an unmatched worker can send $i \in \{1, \ldots, A\}$ applications with probability

$$p_i(\alpha) = \begin{cases} e^{-\alpha} \frac{\alpha^{i-1}}{(i-1)!} & \text{for } i \in \{1, \ldots, A-1\} \\ 1 - \sum_{j=1}^{A-1} p_j(\alpha) & \text{for } i = A, \end{cases}$$

where A is a potentially large but finite integer. I explore two variations. First, following most of the directed search literature, I assume that workers send exactly one application per period, i.e. $A = 1$ and $p_1(\alpha) = 1$ for all α. Subsequently, I relax this assumption and allow for multiple applications, $A > 1$.[12]

Recruitment. At the beginning of the recruitment phase, the productivity x of each potential match is realized, independently across all firm-applicant pairs. An applicant is *unqualified* for the job with probability $1 - q \in [0, 1)$, in which case his productivity x equals zero. Firms can identify unqualified applicants at zero cost, allowing them to eliminate these applicants. With probability q, the applicant is *qualified* and his productivity is a draw from a distribution $Q(x)$ with support $\mathcal{X}$ and finite mean and variance. Firms can learn the exact productivity x of a qualified applicant by interviewing him.

The number of interviews n_R that a firm can conduct depends on its choice of recruiting intensity r. To be precise, I assume that n_R follows a geometric distribution with parameter $\frac{r}{r+1} \in [0, 1]$, such that the probability that a firm can interview $n_R \in \mathbb{N}_1 \equiv \{1, 2, \ldots\}$ applicants is given by $\left(\frac{r}{r+1}\right)^{n_R - 1} \frac{1}{r+1}$ and $\mathbb{E}[n_R|r] = r + 1$.[13] This functional form is convenient because it results in simple expressions for key variables, as I will show in section 2.3. The actual number of interviews taking place equals $\min\{n_Q, n_R\}$, where $n_Q \in \mathbb{N}_0 \equiv \{0, 1, \ldots\}$ is the firm's number of qualified applicants. Since a worker's productivity is unknown ex ante, the firm randomly selects n_R qualified applicants for an interview if $n_Q > n_R$. The firm rejects all applicants which it does not interview or for which the match productivity is revealed to be below its hiring standard.

Matching. In order to understand the mechanism that governs matching, consider unmatched workers and firms with vacancies as nodes in a bipartite network, connected by links representing interviews which revealed a productivity exceeding firms' hiring standards. Firms' preferences over workers are determined by the match productivities x. I assume that workers do not learn match productivity until output is produced, which guarantees that workers' preferences over firms only depend on the *expected* value of employment w, even if the *realized* value of employment varies

[12] The exact functional form is not crucial. I only require i) first-order stochastic dominance of the cumulative distribution in α to make α interpretable as search intensity, and ii) concavity of $\sum_{i=1}^{a} \sum_{j=i}^{A} p_j(\alpha)$ in α for all $a \in \{1, \ldots, A\}$ to guarantee uniqueness of a worker's choice of α. An alternative example of a distribution satisfying these properties is the (truncated) geometric distribution.

[13] $\mathbb{E}[n_R|0] > 0$ eliminates cases without trade, but is otherwise not crucial for the results. The stochastic nature of n_R can be motivated as follows: firms can generally roughly determine the number of applicants that they can interview by hiring more or fewer recruiters; however, the exact number may also depend on factors that could not be anticipated, e.g. some recruiters may unexpectedly not be available or the screening of a candidate takes more or less time than foreseen.

with x. This simplifies the equilibrium analysis considerably.[14] The matching is then assumed to be stable on this network in the sense that no firm matches with an applicant with productivity x while one of its preferred interviewed applicants (i.e. with higher productivity) is hired by another firm at a lower w or remains unemployed. Otherwise, both the firm and the worker could do better by forming a match together. Stability can be motivated by a process in which firms offer their job sequentially to the candidates, and workers are free to reconsider their options.[15] Ties are broken randomly. I denote the job-finding probability of a searching worker by Ψ.

Production. Subsequently, production takes place and payoffs are realized. An endogenous measure $u = (1 - \Psi)\, s$ of workers failed to find a job in the current period and is therefore unemployed. They receive a periodical payoff h—representing the sum of unemployment benefits, value of leisure and household production—which together with their continuation value determines their value of unemployment U_0. Firms that did not fill their vacancies do not produce any output. In contrast, a match between a firm and a worker with productivity x produces a periodical output $x + y$, where y denotes a productivity component common to all matches. The joint match value $M\,(x)$, derived below, is divided between the worker and the firm as prescribed by the contract.

Job Destruction. In the last phase, an exogenous fraction δ of the matches gets destroyed. Employed workers hit by this shock become unmatched. In the next period, they can—like the u unemployed—search for a new job. Hence, a worker who is employed in the current production phase may be employed at a different firm in the production phase of the next period after being unmatched for a very short amount of time. Discreteness of the data may cause this to look like a job-to-job transition, even though the model does not allow for on-the-job search in a classic sense.[16]

2.2 Strategies

Firms' Strategies. Each firm faces two key decisions: whether to enter the market or not and which contract to post. The first decision is straightforward: entry will take place as long as the expected payoff of entry is positive. The optimal contract is less trivial and will be derived below. Denote the equilibrium distribution of contracts by F.

Workers' Strategies. Workers make three decisions: which search intensity to choose, to which contracts to apply, and which job offer to accept. Denote the equilibrium distribution of search

[14] As I discuss in more detail below, this assumption implies that the equilibrium for $A > 1$ has a convenient recursive structure, which is lost otherwise. One can interpret the assumption as a convenient way to model the idea that workers starting a new job may know the expected value of the match, but generally face some uncertainty regarding its exact realization (in reality, for example because of uncertainty regarding match duration or wage growth).

[15] This *deferred acceptance* process, first described by Gale and Shapley (1962), converges in finite time for finite economies. Since the labor market described here contains a continuum of agents, I impose stability by assumption, following Kircher (2009).

[16] This feature of the model is important for the empirical part of this paper where workers' search intensity will be inferred from the total number of workers searching and the total number of applications sent. Menzio and Shi (2011) incorporate actual on-the-job search in a directed search model with one application per period. The combination of simultaneous search and on-the-job search is computationally challenging.

intensities by $H(\alpha)$. Each search intensity translates into a number of applications a. Conditional on this number, a worker solves a portfolio problem to determine the contracts to which he wishes to apply. Denote the solution to this problem by

$$\mathbf{c}_a = (c_{1:a}, \ldots, c_{a:a}) = ((\underline{x}_{1:a}, r_{1:a}, w_{1:a}), \ldots, (\underline{x}_{a:a}, r_{a:a}, w_{a:a})),$$

where, without loss of generality, $w_{1:a} \leq \ldots \leq w_{a:a}$. Although workers send their applications simultaneously, it will occasionally be convenient to refer to the application to $c_{i:a}$ as a worker's i-th application.[17] Conditional on a, let the distribution of workers' application strategies be denoted by G_a.

After receiving job offers, workers take the position that provides them with the best terms of trade; assume—again without loss of generality—that they accept the job offer from the application with the higher index in case of a tie. Let $\xi_{i:a}(\mathbf{c}_a)$ denote the probability that a worker with application portfolio $\mathbf{c}_a$ accepts a job offer from a firm posting $c_{i:a}$, conditional on receiving such a job offer.

2.3 Queue and Matching Probabilities

Queue Length. Workers' application strategies determine the number of applicants of a firm. As well-known in the literature on *urn-ball* models, this number follows a Poisson distribution. The endogenously determined mean of this distribution, the *queue (length)* $\lambda(c)$, equals the ratio of the number of applications sent to c to the number of firms offering c.

A firm does not only care about its number of applicants, but also about their productivity and whether they will accept a potential job offer. Given a queue λ, the number of applicants with productivity greater than $x \in \mathcal{X}$ and without better offers follows a Poisson distribution as well.[18] Its mean $\mu(x; \lambda, \theta)$ equals

$$\mu(x; \lambda, \theta) = \lambda\theta q (1 - Q(x)) \leq \lambda, \tag{1}$$

where θ denotes the endogenous probability that the firm's job offer gets accepted, which depends on the function $\xi_{i:a}(\cdot)$ as well as the distribution of i and a among its applicants.

For what follows, it is useful to define the function $\kappa(x; r, \lambda, \theta)$ as the weighted average of $q\lambda$ and $\mu(x; \lambda, \theta)$, with $\frac{1}{r+1}$ and $\frac{r}{r+1}$ being the respective weights. That is,

$$\kappa(x; r, \lambda, \theta) = \frac{1}{r+1} q\lambda + \frac{r}{r+1} \mu(x; \lambda, \theta). \tag{2}$$

Job-Offer and Hiring Probability. An important role in the equilibrium analysis is played by two outcomes of the matching process: i) a firm's *hiring probability* $\eta(x; r, \lambda, \theta)$, describing the chance that a firm succeeds in hiring a worker with productivity exceeding $x > 0$; and ii) a worker's *job-offer probability* $\psi(\underline{x}, r, \lambda, \theta)$, representing his chances of receiving an offer from a firm to which he

[17] Numbering applications from the lowest w to the highest w is convenient, as it will turn out that workers who only send a few applications will send them to low w.

[18] Lester et al. (2015) discuss this property of the Poisson distribution (which they call *invariance*) in detail.

applied. Despite the complexity of the environment, it turns out that the way in which recruitment is modeled keeps the expressions for these probabilities surprisingly simple, which greatly improves tractability. The following proposition introduces these expressions.[19]

Proposition 1. *Given a hiring standard $\underline{x}$, a recruiting intensity r, a queue length λ, and an acceptance probability θ, a firm hires a worker with productivity exceeding $x \in \mathcal{X}$ with probability*[20]

$$\eta\left(x; r, \lambda, \theta\right) = \begin{cases} \eta\left(\underline{x}; r, \lambda, \theta\right) & \text{for all } x < \underline{x} \\ \frac{\mu(x;\lambda,\theta)}{\kappa(x;r,\lambda,\theta)}\left(1 - e^{-\kappa(x;r,\lambda,\theta)}\right) & \text{for all } x \geq \underline{x}. \end{cases} \tag{3}$$

An applicant to the firm receives a job offer with probability

$$\psi\left(\underline{x}, r, \lambda, \theta\right) = q\left(1 - Q\left(\underline{x}\right)\right) \frac{1 - e^{-\kappa(\underline{x};r,\lambda,\theta)}}{\kappa\left(\underline{x}; r, \lambda, \theta\right)}. \tag{4}$$

If $\kappa\left(x; r, \lambda, \theta\right) = 0$, then, by convention, $\eta\left(x; r, \lambda, \theta\right) = 0$ and $\psi\left(\underline{x}, r, \lambda, \theta\right) = q\left(1 - Q\left(\underline{x}\right)\right)$.

Intuition. To better understand proposition 1, note that $\psi\left(\underline{x}, r, \lambda, \theta\right)$ equals the product of the probability $q\left(1 - Q\left(\underline{x}\right)\right)$ that a worker draws a productivity $x \geq \underline{x}$ and a factor which represents the probability of a job offer, conditional on such a productivity draw. This factor closely resembles the job-offer probability in a standard urn-ball model, except that it depends on $\kappa\left(\underline{x}; r, \lambda, \theta\right)$ instead of the queue length (see e.g. Burdett et al., 2001). In that sense, $\kappa\left(\underline{x}; r, \lambda, \theta\right)$ can be interpreted as the *effective* queue of competitors that a worker faces at a firm, accounting for the possibility of multiple interviews.[21]

To illustrate this, consider two limiting cases. First, when $r \to 0$, all qualified applicants compete for a firm's single interview and job offer, i.e. $\kappa\left(x; 0, \lambda, \theta\right) = q\lambda$. Second, if $r \to \infty$, firms interview all qualified applicants and can make multiple job offers if necessary. Hence, each applicant effectively only faces competition from fellow applicants who exceed the hiring standard and will accept a job offer, i.e. $\kappa\left(x; r, \lambda, \theta\right) \to \mu\left(x; \lambda, \theta\right)$.

These limiting cases also yield intuitive expressions for firms' hiring probability. If $r \to 0$, then $\eta\left(x; 0, \lambda, \theta\right) = \theta\left(1 - Q\left(x\right)\right)\left(1 - e^{-q\lambda}\right)$ for all $x \geq \underline{x}$. This is the product of (in reverse order) the probability that the firm has at least one qualified applicant, the probability that the applicant selected for the interview draws a productivity larger than x, and the probability that this applicant would accept a job offer. If $r \to \infty$, a firm can perfectly rank all its applicants, such that it forms a match with productivity exceeding $x \geq \underline{x}$ as long as it attracts at least one applicant who has such productivity and who is willing to accept a job offer, i.e. $\lim_{r\to\infty} \eta\left(x; r, \lambda, \theta\right) = 1 - e^{-\mu(x;\lambda,\theta)}$.

[19] The proposition describes the probabilities conditional on the composition of a seller's queue. Its scope therefore extends to models with ex ante heterogeneity in worker productivity (e.g. Shimer, 2005a) or models in which applications are sent randomly (e.g. Gautier and Moraga-Gonzalez, 2005; Gautier et al., 2016).

[20] As the $\underline{x}$ only determines the support of η, I suppress it as an argument to keep notation simple.

[21] I borrow this terminology from Kircher (2009) who uses it for μ in his model.

Properties. The following lemma establishes some properties of η which will be convenient in the equilibrium analysis. These properties imply that a firm is more likely to hire a good worker if it attracts a longer queue or chooses a higher recruiting intensity, that the marginal effect is diminishing, and that the queue and recruiting intensity are complements in hiring.

Lemma 1. *The hiring probability η is 1) strictly decreasing in x; 2) strictly increasing in r and λ; 3) strictly concave in r and λ; and 4) strictly supermodular in r and λ.*

Notation and Terminology. The definitions (1) to (4) take the queue length λ and the acceptance probability θ as exogenously given. While this will prove convenient for much of the analysis, both are of course equilibrium objects which depend on a firm's contract c. Hence, a firm's hiring probability equals $\eta\left(x; r, \lambda\left(c\right), \theta\left(c\right)\right)$ in equilibrium. To keep the exposition as simple as possible, I will often slightly abuse notation and refer to this probability as $\eta\left(x; c\right)$. Likewise, I will write $\psi\left(c\right) \equiv \psi\left(\underline{x}, r, \lambda\left(c\right), \theta\left(c\right)\right)$.

Note that the hiring probability $\eta\left(x; c\right)$ conditions on the productivity of a firm's hire. Occasionally, it will be useful to consider the unconditional probability $\eta\left(\underline{x}; c\right)$ that a firm succeeds in filling its vacancy. I will refer to this probability as the *job-filling probability.*

2.4 Value Functions

Value of Unemployment. A worker who is unemployed during the production phase receives a periodical payoff h and can search for a new job in the next period. If we denote the value of search by S, then the value of unemployment equals

$$U_0 = h + \beta S. \tag{5}$$

Value of Search. Given a realization of a number of applications a and an application portfolio $\mathbf{c}_a$, a worker will end up in a position offering terms of trade $w_{i:a}$ with probability $\xi_{i:a}\left(\mathbf{c}_a\right)\psi\left(c_{i:a}\right)$. He chooses $\mathbf{c}_a$ to maximize the expected value of sending a applications,

$$U_a = \max_{\mathbf{c}_a} U_0 + \sum_{i=1}^{a} \xi_{i:a}\left(\mathbf{c}_a\right)\psi\left(c_{i:a}\right)\left(w_{i:a} - U_0\right), \tag{6}$$

Anticipating this, the worker chooses his search intensity α to maximize his value of search,

$$S = \max_{\alpha} \sum_{a=1}^{A} p_a\left(\alpha\right) U_a - k_A \alpha. \tag{7}$$

Recursive Formulation. A worker's acceptance probability must satisfy

$$\xi_{i:a}\left(\mathbf{c}_a\right) = \prod_{j=i+1}^{a}\left(1 - \psi\left(c_{j:a}\right)\right), \tag{8}$$

Substituting this into (6) reveals that—as in Galenianos and Kircher (2009) and Kircher (2009)—we can write workers' payoff in a recursive way. The following lemma formalizes this.

Lemma 2. *The expected value of sending $i \in \{1, \ldots, A\}$ applications satisfies the recursive equation*

$$U_i = \max_c \psi(c) w + (1 - \psi(c)) U_{i-1}. \tag{9}$$

This result implies that a worker's payoff from his i-th application does not depend on the total number of applications that he sends, which considerably simplifies the equilibrium analysis. Going forward, it will often be convenient to interpret U_{i-1} as the outside option associated with a worker's i-th application.

Match Value. A match between a firm and a worker with productivity x produces $x + y$ units of output per period until the job gets destroyed. Job destruction yields the worker his value of search S and the firm a zero payoff because of free entry. Hence, the match value $M(x)$ satisfies the Bellman equation

$$M(x) = x + y + \beta((1 - \delta) M(x) + \delta S). \tag{10}$$

Value of a Vacancy. Given the hiring probability (3), the maximization problem of a firm can be written as $\max_c V(c)$, where $V(c)$ represents the value of creating a vacancy and posting a contract c. That is,[22]

$$V(c) = -\int_{\underline{x}}^{\infty} (M(x) - w)\, d\eta(x; c) - k_V - k_R r.$$

Free entry implies that firms will continue to enter as long as $\max_c V(c)$ is strictly positive.

3 Equilibrium

3.1 Equilibrium Definition

Out-of-Equilibrium Beliefs. Before defining the equilibrium, note that firms must form beliefs regarding the acceptance probability $\theta(c)$ and the queue length $\lambda(c)$ for every $c \in \mathcal{C}$, as they must be able to determine their expected payoff for any feasible contract. While it is straightforward to calculate $\theta(c)$ and $\lambda(c)$ for contracts posted in equilibrium, firms' beliefs off the equilibrium path require extra care. I follow the solution of Galenianos and Kircher (2009), who discuss this issue in detail, by imposing that $\theta(c) = \lim_{\varepsilon \to 0} \theta_\varepsilon(c)$ and $\lambda(c) = \lim_{\varepsilon \to 0} \lambda_\varepsilon(c)$, where $\theta_\varepsilon(c)$ and $\lambda_\varepsilon(c)$ are the acceptance probability and queue length in a perturbed economy in which a fraction ε of firms posts a random contract $c \in \mathcal{C}$, according to some distribution $\widetilde{F}(c)$ with full support.[23] In this economy, workers face a distribution of contracts $F_\varepsilon(c) = (1 - \varepsilon) F(c) + \varepsilon \widetilde{F}(c)$ and will choose application portfolios and search intensities according to $\{G_{1,\varepsilon} \ldots, G_{A,\varepsilon}\}$ and H_ε, respectively. Given these

[22] The minus sign in front of the integral appears because $\eta(x; \cdot)$ represents the probability mass *above* x.

[23] It follows from the results in Galenianos and Kircher (2009) that the exact form of $\widetilde{F}(c)$ does not matter.

distributions, $\theta_\varepsilon(c)$ and $\lambda_\varepsilon(c)$ can be calculated in a straightforward manner for any c, as shown in appendix A.

Equilibrium Definition. I now define an equilibrium as follows.

Definition 1. An equilibrium is a measure of searchers s^*, a measure of vacancies v^*, a distribution of contracts F^*, application distributions $\{G_1^*, \ldots G_A^*\}$, and a search intensity distribution H^*, such that:

1. Optimal Contracts: supp $F^* = \arg\max_{c \in \mathcal{C}} V(c)$;
2. Optimal Entry: $\max_{c \in \mathcal{C}} V(c) \leq 0$ and $v^* \geq 0$, with complementary slackness;
3. Optimal Applications: $\forall a \in \{1, \ldots, A\}$, $\mathbf{c}_a \in$ supp G_a^* if and only if $\mathbf{c}_a$ solves (6);
4. Optimal Search Intensity: $\alpha \in$ supp H^* if and only if α solves (7);
5. Steady State: $s^* = (1 - \Psi^*) s^* + (1 - (1 - \Psi^*) s^*) \delta$;
6. Consistency: $\theta(c)$, $\lambda(c)$ and Ψ^* satisfy the definitions in appendix A.

3.2 One Application

I first analyze the model under the assumption that all workers send one application, i.e. $A = 1$ and $p_1(\alpha) = 1$ for all α. This assumption, which is pervasive in the literature on directed search, simplifies the equilibrium derivation in a number of ways. First, it reduces workers' portfolio problem to the choice of a single contract. Second, it implies that workers will trivially choose a search intensity $\alpha = 0$. Finally, it guarantees that all job offers will be accepted with probability $\theta(c) = \xi_{i:a}(\mathbf{c}_a) = 1$. The resulting environment is a useful starting point to build intuition for some of the economic tradeoffs in the model. As some of the logic is familiar from other directed search models with one application, I keep the exposition brief, focussing on the new elements, and refer to the appendix for a detailed derivation.

Existence and Uniqueness. To find the equilibrium, consider the problem of an individual firm deciding what contract c to post. The firm takes workers' outside option U_0 and the expected payoff U_1 which it must provide to attract potential applicants as given. Hence, for each recruiting intensity r and hiring standard $\underline{x}$, equation (9) acts like a constraint which defines a one-to-one relationship between the firm's terms of trade w and its queue $\lambda(c)$. Substitution of this constraint into the firm's optimization problem yields, after some simplification,

$$\max_{c \in \mathcal{C}} - \int_{\underline{x}}^{\infty} (M(x) - U_0) \, d\eta(x; r, \lambda(c), 1) - \lambda(c)(U_1 - U_0) - k_V - k_R r, \tag{11}$$

where the first two terms represent the expected match surplus and the firms' cost of attracting a queue $\lambda(c)$, respectively.

Because $\theta(c) = 1$, this optimization problem depends on w only through the queue length and can therefore be analyzed in terms of $(\underline{x}, r, \lambda)$ instead of $(\underline{x}, r, w)$. The hiring standard $\underline{x}$ only enters the first term and must therefore solve $M(\underline{x}) = U_0$. That is, only matches with a non-negative match surplus are created. The optimal recruiting intensity r and queue λ are pinned down by the first-order conditions, as (11) is strictly concave in these two variables, by virtue of lemma 1.

Other firms choose the same $\underline{x}$, r and λ. Proposition 2 establishes that there exist unique values of U_0 and U_1 such that these choices are consistent with free entry and equation (5). Given the equilibrium values $(\underline{x}_1^*, r_1^*, \lambda_1^*)$, the terms of trade w_1^* can be obtained from (9), the measure of searchers s^* follows from the steady state condition, and v^* equals $\frac{s^*}{\lambda_1^*}$.

Proposition 2. *For $A = 1$, a unique equilibrium exists. In this equilibrium, all $v^* > 0$ firms that enter the market post the same contract $c_1^* = (\underline{x}_1^*, r_1^*, w_1^*)$, each attracting an equal share of the equilibrium number of searchers s^*, i.e. $\lambda_1^* = \frac{s^*}{v^*}$.*

Productivity Shock. We can now analyze the impact of a one-time, unanticipated shock to aggregate productivity y on steady state outcomes. Proposition 3 presents the effect on the three variables that jointly characterize the matching process, i.e. firms' hiring standard $\underline{x}_1^*$, recruiting intensity r_1^*, and queue length λ_1^*.

Proposition 3. *For $A = 1$, an increase in aggregate productivity y lowers the equilibrium hiring standard $\underline{x}_1^*$, recruiting intensity r_1^*, and queue length λ_1^*.*

The effect of the productivity shock on the queue length is standard: the productivity shock increases the potential gains from trade, which—because firms and workers share these gains in equilibrium—stimulates entry of vacancies, reducing λ_1^*. In a DMP-style model, this is the only effect on the matching process. However, the richness of the model presented here gives rise to two additional channels. First, firms lower their hiring standard, because the productivity shock makes matching with the marginal applicant more attractive relative to remaining unmatched. This prediction is consistent with a large empirical literature documenting the countercyclicality of hiring standards (see Sedláček, 2014, for an overview). Second, firms adjust their recruiting intensity. The direction of this change is less obvious, as a productivity shock has two opposing effects: i) the increase in y and the corresponding reduction of $\underline{x}_1^*$ increase the potential gains from interviewing, causing firms to choose a higher r_1^*, but ii) the shorter queue λ_1^* provides firms with an incentive to lower their recruiting intensity, since r and λ are complements in hiring, as established in lemma 1. Given the structure of the model, it turns out that the latter effect dominates. Whether this is consistent with data remains an open question; to the best of my knowledge, no empirical evidence is available on the cyclicality of firms' interviewing efforts.

To analyze the effect of the productivity shock on firms' job-filling probability $\eta_1^* \equiv \eta(\underline{x}_1^*; r_1^*, \lambda_1^*, 1)$, note that

$$\frac{d\eta_1^*}{dy} = \underbrace{\frac{\partial \eta_1^*}{\partial x}\frac{d\underline{x}_1^*}{dy}}_{>0} + \underbrace{\frac{\partial \eta_1^*}{\partial r}\frac{dr_1^*}{dy}}_{<0} + \underbrace{\frac{\partial \eta_1^*}{\partial \lambda}\frac{d\lambda_1^*}{dy}}_{<0},$$

where the first and second factor in each term are characterized by lemma 1 and proposition 3, respectively.[24] As standard, the decrease in the queue length λ_1^* means that there are fewer searchers per vacancy, causing firms' job-filling probability to go down. The reduction of firms' recruiting intensity r_1^* further decreases their chances of hiring, while the relaxation of the hiring standard $\underline{x}_1^*$ increases their chances of forming a match. Depending on parameter values, these two opposing effects either amplify or mitigate the effect that operates through the queue length. Intuitively, the positive effect of the change in the hiring standard vanishes when the productivity distribution has little mass around $\underline{x}_1^*$, while the negative effect that operates through the recruiting intensity disappears when arbitrarily small or large recruiting costs cause firms to always choose $r_1^* = 0$ or $r_1^* \to \infty$, respectively. The following proposition formalizes this.

Proposition 4. *For $A = 1$, $\frac{\partial \eta_1^*}{\partial x} \frac{d\underline{x}_1^*}{dy} \to 0$ if $Q'\left(\underline{x}_1^*\right) \to 0$, while $\frac{\partial \eta_1^*}{\partial r} \frac{dr_1^*}{dy} \to 0$ if $k_R \to 0$ or $k_R \to \infty$.*

Viewed through the lens of a standard matching function, the effects that operate through r_1^* and $\underline{x}_1^*$ present themselves as changes in match efficiency. Recent evidence that match efficiency is procyclical in nature (Davis et al., 2013; Barnichon and Figura, 2015; Sedláček, 2014, 2016) is then consistent with the case in which the effect of the hiring standard dominates.

3.3 Multiple Applications

I now return to the full model by re-introducing the possibility for workers to send more than one application ($A > 1$), assuming throughout that the search cost k_A is sufficiently small such that they indeed choose $\alpha > 0$.[25] This change affects firms' recruiting decisions through workers' endogenous rejection of job offers. As I will discuss, equilibrium in this case features cross-sectional variation in recruitment decisions and outcomes.

Contract Dispersion. Simultaneous search complicates the equilibrium analysis considerably, as it makes the acceptance probability $\theta\left(c\right)$ an endogenous variable. Fortunately, one can build on the insights provided by Galenianos and Kircher (2009) and Kircher (2009). They prove for related environments—but without productivity differences, recruiting intensities and hiring standards—that equilibrium must have a very specific structure. In particular, if workers send at most A applications, A different values for w emerge in equilibrium, $w_1^* < w_2^* < \ldots < w_A^*$, and workers apply at most once to each value. The following lemma extends these results to the environment of this paper.

Lemma 3. *For $A > 1$, any equilibrium gives rise to A different contracts, $c_1^*, \ldots, c_A^*$, each of them associated with a different hiring standard, $\underline{x}_1^* < \ldots < \underline{x}_A^*$, and different terms of trade, $w_1^* < \ldots < w_A^*$. Workers apply at most once to each contract.*

Key to understanding lemma 3 is the recursive relationship (9), which reveals two important facts about a worker's marginal rate of substitution (MRS) between the terms of trade w and the

[24] See the proofs in the appendix for explicit expressions for each derivative on the right-hand side.

[25] Alternatively, one could assume that $p_1'\left(\alpha\right) \to -\infty$.

job-offer probability ψ when sending his i-th application. First, the MRS does not depend on his total number of applications a, which means that one can abstract from heterogeneity along this dimension for the purpose of this lemma. Second, the MRS varies with i; in particular, the worker is willing to accept a larger decline in ψ in exchange for higher w while sending his j-th application than while sending his i-th application, for all $j > i$, because the outside option U_j^* is strictly better than U_i^*.

Optimality of workers' application behavior implies that a firm posting a contract c expects to attract a queue length $\lambda(c)$ which is consistent with the lower envelope of the A indifference curves, as illustrated in the top plot of figure 2 for $A = 2$. The terms of trade corresponding to the $A-1$ kinks in this upper envelope are denoted by $\overline{w}_1, \ldots \overline{w}_{A-1}$ and divide the contract space into A segments, $\underline{\mathcal{C}}_i \equiv \mathcal{X} \times \mathcal{R} \times [\overline{w}_{i-1}, \overline{w}_i)$, where $\overline{w}_0 = 0$ and $\overline{w}_A = \infty$.[26]

Consider now the problem of a firm considering to post a contract in segment $\underline{\mathcal{C}}_i$. The corresponding part of the upper envelope acts like a constraint which defines a one-to-one relationship between the firm's terms of trade $w \in [\overline{w}_{i-1}, \overline{w}_i)$ and its queue $\lambda(c)$, for each recruiting intensity r and hiring standard $\underline{x}$. Substitution of this constraint into the firm's optimization problem yields, after some simplification,

$$\max_{c \in \underline{\mathcal{C}}_i} - \int_{\underline{x}}^{\infty} \left(M(x) - U_{i-1}\right) d\eta\left(x; r, \lambda(c), \theta(c)\right) - \lambda(c)\,\theta(c)\left(U_i - U_{i-1}\right) - k_V - k_R r. \tag{12}$$

This expression resembles (11) in a number of ways, but there is also an important difference: the acceptance probability $\theta(c)$ is not necessarily equal to 1. Fortunately, however, the insight of Galenianos and Kircher (2009) that $\theta(c)$ must be constant within a segment, i.e. $\theta(c) = \theta_i$ for all $c \in \underline{\mathcal{C}}_i$, applies here.[27]

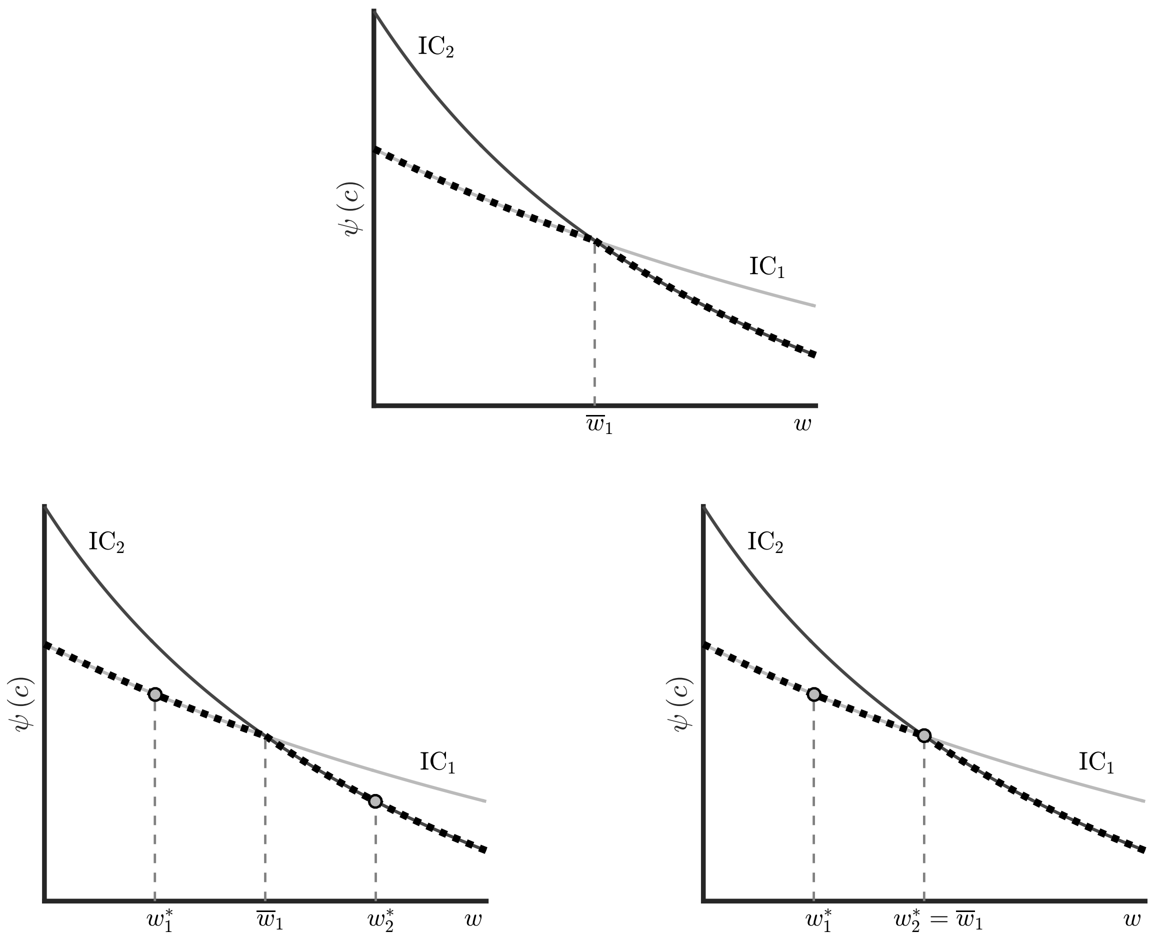
As a result, (12) depends on w only through the queue length, so one can analyze it in terms of $(\underline{x}, r, \lambda)$ instead of $(\underline{x}, r, w)$, as for $A = 1$. The hiring standard $\underline{x}$ only enters the first term and must therefore solve $M(\underline{x}) = U_{i-1}$. That is, firms posting better terms of trade are more selective in order to reduce the number of times that a worker turns down a more productive job for a less productive job. Further, a unique solution for λ and r exists in each segment, because (12) is strictly concave in these two variables.

Existence. Lemma (3) goes a long way to characterizing equilibrium. The final step is to show that workers' search intensity and application behavior given the contracts $c_1^*, \ldots, c_A^*$ can be consistent with the cutoffs $\overline{w}_1^*, \ldots \overline{w}_{A-1}^*$ that imply these contracts. Proposition 5 uses Brouwer's fixed point theorem to establish existence of an equilibrium in which workers sending $a \in \{1, \ldots, A\}$ applications apply to the a contracts with the least favorable terms of trade, resembling equilibrium application

[26] The boundaries between the segments require extra care. The proof of lemma 3 addresses this.

[27] Galenianos and Kircher (2009) prove this formally using trembles in firms' behavior. A more informal—but perhaps more intuitive—argument is the following: the probability $\theta(c)$ with which an applicant accepts a job offer at $c \in \underline{\mathcal{C}}_i$ depends on where he sends subsequent applications. This decision depends on his outside option for these applications, which is determined by the expected payoff that he gets from c. Since this expected payoff is the same for all contracts in $\underline{\mathcal{C}}_i$, the acceptance probability must be the same too.

Figure 2: Application Behavior and Two Types of Equilibrium.



IC$_i$ = indifference curve implied by (9) for $i \in \{1, 2\}$. Lower envelope (dashed) = relation between w and $\psi(c)$ implied by workers' optimal application behavior.

behavior in Kircher (2009).

Proposition 5. *For $A > 1$, an equilibrium exists in which workers sending $a \in \{1, \dots, A\}$ applications apply to $(c_1^*, \dots, c_a^*)$.*

Uniqueness. The results of Galenianos and Kircher (2009) and Kircher (2009) regarding uniqueness of the equilibrium do not necessarily carry over to the more general environment considered here.[28] The reason lies in the behavior of workers who send less than the maximum number of applications and is easiest understood for the case $A = 2$, which is illustrated in the bottom two plots of figure 2. Consider the equilibrium described in proposition 5, in which workers applying once send their application to c_1^* with probability $\gamma = 1$. If the terms of trade w_2^* in the contract c_2^* are strictly larger than the cutoff $\overline{w}_1$ in this equilibrium (bottom left plot), this strategy is the unique solution to the optimization problem of these workers. However, as I will show below, the first-order conditions will imply terms of trade below the cutoff $\overline{w}_1$ for some parameter values. When that is the case, the corner solution $w_2^* = \overline{w}_1$ arises and workers applying once are indifferent between c_1^* and c_2^*, allowing them to randomize between these contracts with arbitrary probability (bottom right plot). Since firms are indifferent between both contracts as well, an indeterminacy arises which leads to multiplicity of the equilibrium.

To see this, suppose that firms change their beliefs about the application behavior of workers applying once and expect them to apply to c_1^* with probability γ slightly below 1 and to c_2^* with probability $1-\gamma > 0$. Everything else equal, more firms will then offer c_2^* instead of c_1^* to cater to the additional applications expected at c_2^*. Workers observe this and will conclude that the submarket formed by c_2^* has become more attractive. Workers applying twice must continue to spread their applications across the two contracts, but workers applying once will adjust their application behavior, automatically fulfilling firms' new beliefs. In other words, there exists a strategic complementarity between the choice of submarket by firms and workers applying once.

Of course, this intuition is not fully complete, because the change in firms' beliefs does not only alter the relative mass of agents in each submarket, but also the optimal contracts c_1^* and c_2^* themselves. The reason is the following. When $A = 2$, the acceptance probability for firms posting c_1^* equals

$$\theta\left(c_1^*\right) = \frac{p_2\left(\alpha^*\right)\left(1 - \psi\left(c_2^*\right)\right) + \gamma p_1\left(\alpha^*\right)}{p_2\left(\alpha^*\right) + \gamma p_1\left(\alpha^*\right)}. \tag{13}$$

A lower value for γ means that a larger fraction of the applications to c_1^* now comes from workers who apply twice and may reject an offer. As a result, $\theta\left(c_1^*\right)$ decreases and it becomes harder for the firms posting this contract to hire.[29] In other words, the frictions in this submarket worsen. This makes it less attractive to create vacancies in this submarket, which increases the expected queue length and changes the contract c_1^* that firms choose to post. All of this affects worker's expected

[28] Galenianos and Kircher (2005) conjecture on the basis of numerical simulations that "there may be multiplicity of equilibria" if some workers send N applications and other workers send $N + 1$ applications, but do not provide a characterization.

[29] The acceptance probability $\theta\left(c_2^*\right)$ of course remains equal to 1.

payoff U_1^* from applying in this submarket, which in turn changes the cutoff $\overline{w}_1$ as well as the optimal contract c_2^* in the second submarket. This feeds back again into the first submarket through $\psi(c_2^*)$ in (13) as well as workers' value of search S^* which affects U_0^*.

Importantly, however, these changes are all continuous in γ. As long as the change in γ is small enough, the optimal terms of trade w_2^* will therefore continue to be a corner solution, at the new value for $\overline{w}_1$, which justifies beliefs $\gamma < 1$. Hence, any γ sufficiently close to 1 is consistent with equilibrium in this case and the multiplicity arises.

When $A > 2$, workers' indifference set may become bigger. For example, if $A = 4$ and $w_i^* = \overline{w}_{i-1}$ for $i \in \{2, 3, 4\}$, a worker sending three applications is exactly indifferent between each of the *four* application portfolios that consist of three different contracts; similarly, workers applying $i \in \{1, 2\}$ times are indifferent between $i+1$ different application portfolios. That is, instead of one free parameter, there now exist $\frac{A(A-1)}{2} = 6$ degrees of freedom. To keep notation manageable, I will focus on the extremes. That is, when a worker sending a applications is indifferent between different application portfolios, I assume that he will choose the one with the lowest terms of trade—$(c_1^*, \ldots, c_a^*)$ in the example—with probability γ and the one with the highest terms of trade—i.e. $\left(c_2^*, \ldots, c_{a+1}^*\right)$—with probability $1 - \gamma$. For the reasons explained above, this suffices to obtain a continuum of equilibria, although many other equilibria exist, involving the application portfolios (c_1^*, c_3^*), (c_1^*, c_2^*, c_4^*) and (c_1^*, c_3^*, c_4^*). Lemma 4 summarizes these results.

Lemma 4. *For $A > 1$, let $\widehat{w}_i$ denote the terms of trade implied by the first-order conditions of a firm considering to post a contract in $\underline{\mathcal{C}}_i$. If there exists an equilibrium in which $\widehat{w}_i < \overline{w}_{i-1}$ for some $i \in \{2, \ldots, A\}$, then a continuum of other equilibria exists.*

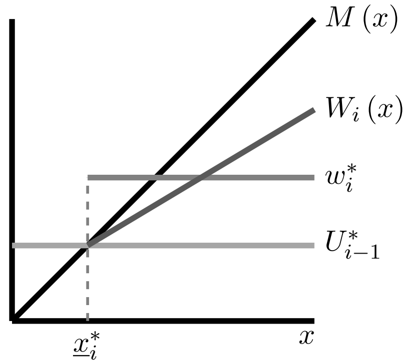
Lemma 4 establishes that a continuum of equilibria exists if $\widehat{w}_i < \overline{w}_{i-1}$ for some $i \in \{2, \ldots, A\}$. Since this is a condition on endogenous variables, it is not a priori obvious whether this condition ever holds. However, it turns out that it does; building on the insights developed by Galenianos and Kircher (2009) and Kircher (2009), one can show that for certain values of the remaining parameters the condition is satisfied for sufficiently large k_R but violated for sufficiently small k_R. Proposition 6 formalizes this.

Proposition 6. *A unique equilibrium exists for k_R sufficiently small. In contrast, a continuum of equilibria exists if the recruiting cost k_R is sufficiently large and workers are sufficiently homogeneous, i.e. $q \to 1$, $Q(x)$ degenerate, and $k_A \to 0$ such that $p_A(\alpha) \to 1$.*

The conditions in proposition 6 are sufficient but not necessary; the complexity of the model unfortunately prevents a precise characterization of conditions on the parameters of the model which are both necessary and sufficient for the equilibrium (not) to be unique. In section 4, I will therefore analyze uniqueness of the calibrated equilibrium numerically.

Contracts. So far, I have remained agnostic about the precise way in which firms implement their terms of trade w_i^*. Of course, the easiest implementation would be a constant wage, independent of productivity, for the entire duration of the match, such that workers' realized value of employment

Figure 3: Wage Menus.



$W_i(x)$ is exactly w_i^* for all $x \geq \underline{x}_i^*$. This implementation does not require workers to be uninformed about match productivities in the matching phase (as this information would be irrelevant for their payoffs), but relies on a strong form of commitment: in equilibrium, a firm posting c_i^* would ex post prefer not to hire a worker with productivity $\underline{x}_i^*$ at this wage, since $W_i(\underline{x}_i^*) = w_i^* > U_{i-1}^* = M(\underline{x}_i^*)$.

This raises the question whether there exist alternatives that require less commitment. Such alternatives must satisfy three conditions. First, the expectation of $W_i(x)$ must equal w_i^*. Second, individual rationality ex post for firms and workers requires $U_{i-1}^* \leq W_i(x) \leq M(x)$ for all $x \geq \underline{x}_i^*$. Third, making sure that firms prefer to hire the most productive applicant requires $W_i'(x) \leq M'(x)$. While various solutions to these conditions exist, perhaps the simplest one is a menu of wages which provides workers with their outside option U_{i-1}^* plus an appropriately chosen fraction of the difference $M(x) - U_{i-1}^*$. Figure 3 illustrates this idea.[30]

Cross-Sectional and Cyclical Variation. Although proposition 3 describes the cross-sectional variation in hiring standards across terms of trade, it is silent regarding recruiting intensities and queue lengths. The cross-sectional patterns for those variables cannot readily be characterized and may depend on parameter values as well as the equilibrium selection rule. The same is true for the response of firms' recruiting choices with respect to a productivity shock; although the effects described in section 3.2 are—ceteris paribus—still present in each of the submarkets, a full characterization requires not only aggregation across submarkets, but also accounting for the change in workers' search intensities, their acceptance probabilities and the set of cutoffs between the segments, all of which affect firms' choices. To make progress, I will therefore calibrate the model in the next section.

[30]For these alternative contracts, the assumption that workers are uninformed about match productivities in the matching phase is important for $A > 1$. Without this assumption, workers may prefer a job offer $W_{i-1}(x)$ over a job offer $W_i(x')$ simply because x is sufficiently higher than x'. This violation of the recursive structure of workers' application problem complicates the equilibrium analysis significantly and is left for future work.

4 Calibration

I calibrate the model in two steps.[31] First, some parameters are set exogenously, after which the remaining parameters are chosen to match aggregate data from the Current Population Survey (CPS) and the EOPP data mentioned in the introduction. The EOPP data is particularly suitable in the context of my model since it contains more detailed information on firms' recruiting decisions than other available data sets. A caveat is that the EOPP data covers firms hiring for low-skilled jobs in 1980–1981, which means that any inference drawn from it does not necessarily hold for other samples. I provide additional details regarding the data and the calibration in the online appendix.

4.1 Exogenous Parametrization

Period Length and Discounting. Calibrating the model requires choosing a period length. In theory, the data contains some information on this parameter because it affects various equilibrium outcomes. For example, workers sending two applications per two weeks yields wage dispersion and rejection of job offers, while one application per week does not. Further, as in Moen (1999), firms being able to compare and rank multiple applicants makes a worker's probability of being hired an increasing function of his productivity, unlike when firms meet at most one applicant at a time.[32] Nevertheless, identification of the period length along these dimensions cannot be established easily and would be very indirect at best. In line with most of the literature (e.g. Menzio and Shi, 2011), I therefore exogenously fix the period length to one month.[33] Both firms and workers discount future payoffs at a factor $\beta = 0.96^{1/12}$ per period.

Search, Matching and Job Destruction. As I describe in the online appendix, CPS data can be used to calibrate the job-finding probability Ψ and the separation probability δ. This yields $\Psi = 0.412$ and $\delta = 0.059$.[34] These values—combined with the steady state assumption—imply that a mass $s = 0.132$ of workers applies to jobs in each search phase. I assume that they send at most $A = 15$ applications, which will prove not to be restrictive given the equilibrium value of α.

Productivity. The distribution $Q(x)$ is assumed to be log-normal with scale parameter normalized to 0 and shape parameter σ, so median productivity is 1. The common productivity component y is

[31] By performing one aggregate calibration, I implicitly assume that workers are ex-ante identical or that labor markets at different skill levels are scale replicas of each other. A previous draft Wolthoff (2012) formalized the latter assumption to additionally use wage data in a Maximum Likelihood estimation of a simpler version of the model. The estimates for key parameters (e.g., search and recruiting costs) were similar to the values presented here. In reality, these parameters may of course vary with skill, but the EOPP data is not rich enough to identify this variation.

[32] In addition to the discussion in Moen (1999), see also Wolthoff (2014) for an analysis of period lengths.

[33] This value is shorter than the 3-4 months documented in van Ours and Ridder (1992, 1993) using Dutch data, but long enough for most firms in the EOPP data to complete their screening, as I will show below.

[34] Note that δ cannot directly be compared to the 'employment-exit probability' of Shimer (2012). First, δ only describes a transition from employment to the pool of searchers; unemployment only follows in case that search fails. Second, Shimer (2012) needs to control for time aggregation because he uses a continuous-time model, whereas here the period length in the model corresponds to the frequency of the data (1 month).

normalized to 0 in the baseline calibration, but takes the value 0.01 when analyzing the elasticities of the equilibrium outcomes with respect to a productivity shock.

Equilibrium Selection. As I will discuss below, a multiplicity of equilibrium arises for the calibrated parameter values since $w_i^* = \overline{w}_{i-1}$ for all $i \in \{2, \dots, A\}$. I present the results for two different equilibrium selection rules, $\gamma = 0$ and $\gamma = 1$. That is, when workers are indifferent between multiple application portfolios, they apply to the one with either the highest ($\gamma = 0$) or the lowest ($\gamma = 1$) terms of trade.[35]

4.2 Matching Moments

Parameters and Moments. After making the above assumptions, six parameters (k_V, k_R, k_A, h, q, and σ) remain for which no direct evidence regarding their value is available. I calibrate these parameters by targeting the following six data moments: 1) a monthly job-finding probability of 0.412 for a worker; 2) a monthly job-filling probability of 0.884 for a firm; 3) an average of 9.69 applicants per firm, conditional on hiring; 4) an average of 5.70 interviews per firm, conditional on hiring; 5) an average of 11.01 hours of managerial time spent on recruiting, conditional on hiring; and 6) an elasticity of unemployment with respect to productivity of -9.5. The value of time spent on recruiting is a proxy for firms' total recruiting cost $k_R r$, while the other five moments have direct counterparts in the model. As explained above, the first moment follows from the CPS. Shimer (2005b) uses the same data source to calculate the last moment. The remaining four moments can be calculated from the EOPP data, as I describe in the online appendix.

Intuition for Identification. Although the moments jointly determine the parameter values, it might be useful to briefly consider which moments are particularly informative of a certain parameter to develop some intuition for identification. To start, note that workers' matching and firms' job-filling probability together pin down the equilibrium measure of vacancies.[36] By standard logic, the higher this measure is, the lower the entry cost k_V must be. Given the measure of searchers and vacancies, firms' number of applicants then has implications for workers' search intensity, which in turn is informative of their search cost k_A.

Further, as pointed out by Hagedorn and Manovskii (2008), the fact the elasticity of unemployment is high suggests that match surplus is small, which means that h must be high. A high value of h causes firms to set hiring standards that are high as well. Given these hiring standards and firms' number of interviews, the standard deviation of productivity σ determines firms' job-filling probability, as it controls the likelihood that an interviewee will be "below the bar." The fact that firms' job-filling probability is high therefore indicates that σ must be relatively small, which means that the gains from interviewing are limited. As a result, firms will choose a relatively low recruiting

[35] Intermediate values of γ yield results that lie between these two extremes.

[36] Aggregate consistency requires that $0.884v = \Psi s = 0.412 \times 0.132 = 0.054$, which implies $v = 0.061$.

Table 1: Calibrated Parameter Values.

	$\gamma = 0$	$\gamma = 1$
Entry cost k_V	0.279	0.394
Recruiting cost k_R	0.010	0.008
Search cost k_A	0.036	0.041
Payoff from unemployment h	0.892	0.888
Fraction of qualified applicants q	0.738	0.690
Shape parameter σ	0.036	0.048

intensity. The smaller firms' recruiting intensity is, the higher q and k_R must be to explain the data moments regarding the number of interviews and the total cost of recruiting, respectively.[37]

Calibrated Values. Despite the high degree of non-linearity, the six parameters can match the six moments perfectly. This fact is not obvious and adds credibility to the model with multiple applications. After all, a model with $A = 1$ cannot simultaneously match these moments, since 9.69 applicants or 5.70 interviews per firm imply upper bounds on workers' job-finding probability equal to 0.10 and 0.18, respectively. The solution of the calibration is displayed in table 1.

As this table reveals, the calibrated parameters are reasonably similar for the two different selection rules.[38] An exception is the entry cost k_V which is higher when $\gamma = 1$. The reason is straightforward: as discussed above, the frictions in the matching process are less severe when $\gamma = 1$, but the data moments imply the same measure of vacancies for both values of γ; reconciling both facts requires that the entry cost is higher for $\gamma = 1$. Given that the outcomes are otherwise quite similar, I will focus on $\gamma = 1$ hereafter, unless explicitly mentioned otherwise.

The periodical payoff from unemployment h is 0.888, which is higher than the value in Hall and Milgrom (2008), but lower than the value in Hagedorn and Manovskii (2008).[39] Each unit of search intensity costs a worker approximately 4.6% of h. The recruiting cost k_R is considerably lower at 0.8% of the monthly output created by the median qualified applicant. Hence, the reason for the fact that firms do not interview all applicants does not primarily lie in recruiting being very costly, but in a substantial fraction of applicants (approximately 30%) not being qualified for the job.[40] Among the workers who are qualified, the standard deviation of productivity is almost 5%.

4.3 Equilibrium Outcomes

Table 2 presents equilibrium outcomes for the above parameter values. Column I and II display the levels of various endogenous variables, whereas column III and IV report their elasticities with

[37] Column I and II of table 3 illustrate this logic by presenting results for two different values of h.

[38] Note that this fact is not informative of the impact of a change in the equilibrium selection rule for given parameter values. I address that question in section 4.4.

[39] I analyze the degree of amplification that my model is able to generate in section 4.4.

[40] While no representative data on q is available, informal evidence suggests even higher numbers. For example, Weber (2012) writes: "Most recruiters report that at least 50% of job hunters don't possess the basic qualifications for the jobs they are pursuing."

Table 2: Equilibrium Outcomes and Elasticities.

	(I)	(II)	(III)	(IV)
	Equilibrium values		Elasticities	
	$\gamma = 0$	$\gamma = 1$	$\gamma = 0$	$\gamma = 1$
Aggregate variables				
Unemployment rate	0.077	0.077	−9.5	−9.5
Measure of vacancies	0.061	0.061	0.8	0.8
Labor market tightness	0.793	0.793	11.4	11.3
Search variables				
Search intensity	3.47	3.44	10.0	10.2
Avg. number of applications	4.47	4.44	7.8	7.9
Avg. search time (hours)	23.13	26.54	10.0	10.2
Recruiting variables				
Avg. number of applicants	9.59	9.54	1.3	1.5
Avg. recruiting intensity	10.62	13.95	1.1	1.2
Avg. number of interviews	5.30	5.32	1.2	1.4
Avg. recruiting time (hours)	11.01	11.00	1.1	1.2
Avg. hiring standard	0.993	0.993	−0.1	−0.1
Avg. offered surplus share	0.460	0.431	0.2	0.5
Avg. acceptance probability	0.806	0.807	−1.8	−1.8
Avg. number of offers	1.11	1.10	2.0	2.0
Matching variables				
Job-finding probability	0.412	0.412	6.4	6.4
Job-filling probability	0.884	0.884	−0.0	0.0
Avg. match output	1.042	1.056	−0.0	−0.0

respect to match output $x + y$.[41] I now discuss some of these outcomes in more detail.

Aggregate Variables. The unemployment rate implied by the model is $u = 0.077$, which is close to the one observed in the data (0.078), taking into account the age structure of the sample. Hence, the steady assumption is reasonable despite the recessive state of the US economy in 1980 and 1981. A fraction $\delta\Psi = 0.024$ of the employed workers moves from one job to the next without intermittent unemployment spell. This value is consistent with monthly job-to-job transition rates reported by Fallick and Fleischman (2004), Moscarini and Thomsson (2007) and Nagypal (2008), which range from 2.2% to 3.2%. The measure of vacancies v is 0.061, which is somewhat higher than the value

[41] As mentioned above, I calculate these elasticities by increasing y from 0 to 0.01. The corresponding change in average match output $x + y$ among employed workers is less than 1% (i.e. 0.93% for $\gamma = 0$ and 0.94% for $\gamma = 1$) for two reasons. First, firms' screening of applicants implies that average match productivity x is larger than 1 (see column I and II). Second, firms relax their hiring standards $\underline{x}$ after the productivity shock, causing average match productivity x to go down. The magnitude of the latter effect turns out to be small (see column III and IV); a similar effect plays a more important role in Menzio and Shi (2011).

in Davis et al. (2013) for the relevant time period.[42] This difference seems to merely reflect that my definition of a vacancy is wider than theirs, since any match in the model requires a preceding vacancy. The volatility of v is low, precisely because the volatility of u is high; the strong decline in unemployment reduces firms' incentives to create vacancies, offsetting the positive effect of the productivity shock on entry. As pointed out by Menzio and Shi (2011), adding on-the-job search may provide a partial solution for this. The volatility of labor market tightness v/u is nearly 11.5, which is 60% of the value reported by Shimer (2005b).

Search Variables. Workers choose a search intensity α which allows them to send 4.4 applications on average. If we assume that a year consists of 50 weeks of 5 days of 8 productive hours, the time workers spend on search can be calculated as $\frac{2000}{12}\frac{\alpha k_A}{h}$, which amounts to 26.5 hours. No estimates of this number are available in the literature for the workers in my sample, but the values I find are a bit higher than the results of Krueger and Mueller (2012) who document that unemployed workers in the US spend approximately 16 hours a month on job search. Dividing total search time by the number of applications implies that the average time cost of an application is 6 hours.

The elasticity of search intensity is 10, which is roughly consistent with the evidence on the volatility of search intensity in response to aggregate conditions provided by Krueger and Mueller (2010). Using time-use data and cross-state variation in unemployment benefits, they find an elasticity of time spent on job search with respect to benefits between -1.6 and -2.2. My model yields a value of -2.4 if I assume that the benefit component of h equals 0.25, which is the value used by Hall and Milgrom (2008) and Mitman and Rabinovich (2014).[43]

Recruiting Variables. The average firm receives 9.5 applications.[44] It conducts just over 5 interviews, corresponding to roughly 80% of its number of qualified applicants. In other words, the frictions in the recruiting process are limited. Dividing recruiting time by the number of interviews implies that the average time cost of an interview equals almost 2.1 hours. Firms set hiring standards around 0.99, which means that on average 44% of the interviewed workers are rejected because they are too unproductive. The average offer promises the worker terms of trade corresponding to approximately 43% of the expected match surplus and is accepted with probability 0.81. Consequently, the average number of job offers a firm makes is 1.10. This number almost exactly coincides with the sample average in the EOPP data (see the online appendix), even though it was not targeted in the calibration.

Firms' number of applicants slightly increases in response to the productivity shock. While this may seem counterintuitive, it is consistent with the limited evidence that is available; Marinescu

[42]Roughly 3%, based on an approach which combines the Conference Board's Help-Wanted Index with the Job Openings and Labor Turnover Survey (JOLTS).

[43]See Kaas (2010) for a similar exercise. Note that h and y enter the equilibrium conditions only through the sum $x + y - h$, so increasing benefits has qualitatively the same effect as decreasing y.

[44]Not surprisingly, this number is (slightly) smaller than the number of applicants conditional on hiring, used as target in the calibration, since firms with more applicants are (slightly) more likely to hire.

(2015) finds that firms receive fewer applications when the potential gains from trade are small.[45] Recall from the discussion after proposition 3 that a productivity shock affects firms' recruiting intensity directly as well as through the number of applicants. With both these effects now pointing in the same direction, recruiting intensity becomes procyclical. Not surprisingly, the hiring standard and the acceptance probability both go down in response to the change in productivity.

Matching Variables. The elasticity of workers' job-finding probability with respect to productivity is approximately 6.4, which is close to the elasticity of 5.9 that Shimer (2005b) reports for the US. The corresponding elasticity for firms' job-filling probability is very close to 0. Two factors contribute to this. First, the procyclicality of recruiting intensity mitigates the negative effect of a productivity shock on firms' job-filling probability. Second, the concavity of η implies that job-filling probabilities close to the upper bound of 1, as is the case here, are simply not very elastic; this suggests that allowing firms to hire multiple workers as in Kaas and Kircher (2015) would be fruitful for further improving the performance of the model.

Cross-Sectional Variation. Since firms' outcomes depend on the contract that they post, there exists substantial cross-sectional variation around the averages discussed above. The plots in figure 4 illustrate this. The top left plot displays the fraction of firms posting particular terms of trade, expressed as a share of the expected match surplus to ease interpretation. This surplus share varies between approximately 0.37 for firms posting c_1^* and 0.52 for firms posting c_{15}^*. Note that the plot highlights the key difference between the two equilibrium selection rules: if $\gamma = 1$, many firms post c_1^* since all workers apply to this contract, while very few firms offer c_1^* if $\gamma = 0$, because it only attracts workers sending 15 applications in this case.

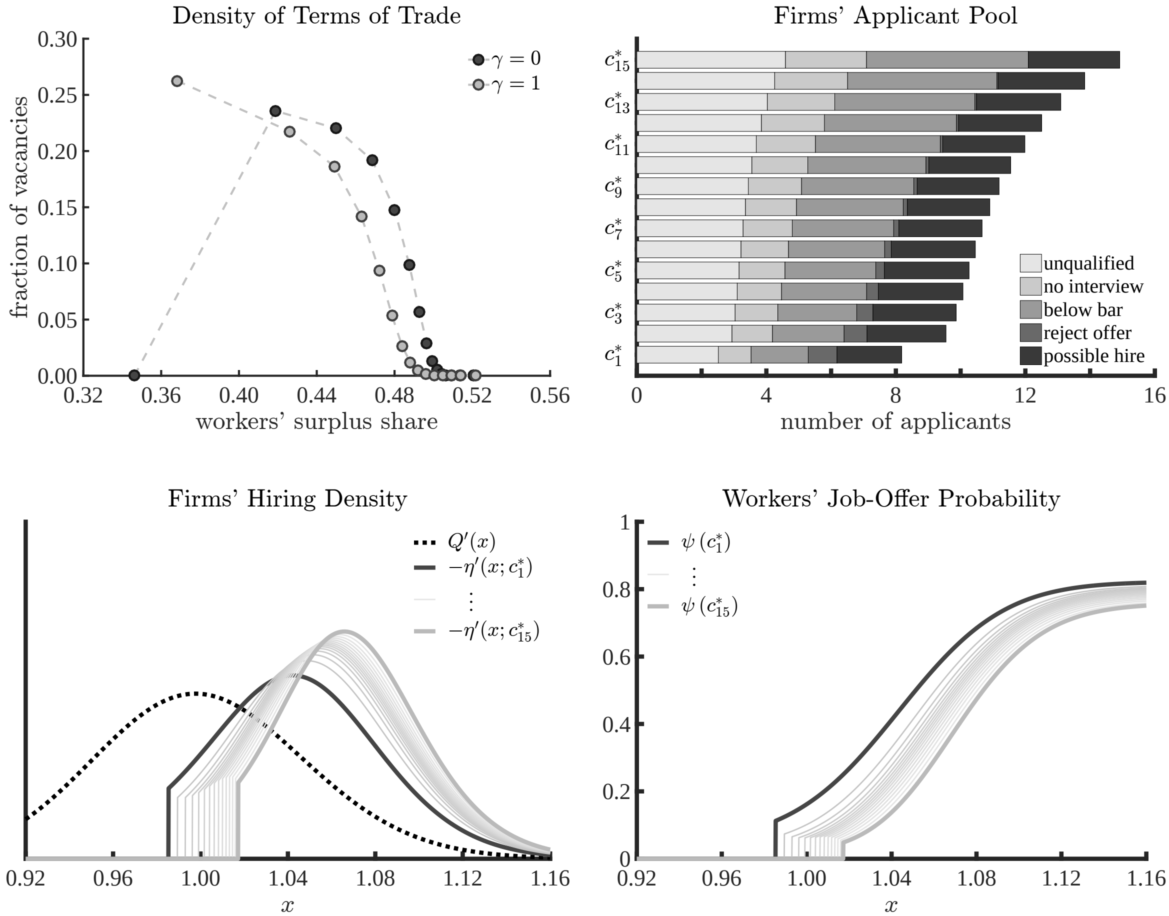
Except for this difference, the equilibrium outcomes are very similar for the two values of γ. The remaining plots in figure 4 therefore focus again on $\gamma = 1$. The top right plot shows firms' applicant pools conditional on their choice of contract. Firms posting better terms of trade receive more applications and conduct more interviews, although they interview a slightly smaller *fraction* of their (qualified) applicants.[46] These firms further set higher hiring standards—causing more applicants to be "below the bar"—but see their offers rejected less often, leaving a larger number of potential hires.

The bottom left plot in figure 4 displays the likelihood $-\frac{\partial}{\partial x}\eta\left(x; c_i^*\right)$ that a firm posting c_i^* hires a worker with productivity x. A few things stand out. First, firms' terms of trade are positively correlated with their job-filling probability, as represented by the mass under the graph. For example, a firm posting c_1^* matches with probability 0.85, while a firm posting c_{15}^* hires with probability 0.89; this relation is—at least in a qualitative sense—consistent with the link between recruiting choices and job-filling probabilities described by Davis et al. (2013). Second, firms that offer better terms of

[45] Identification in her paper comes from extensions in the duration of unemployment insurance benefits. Interpreting this as an increase in h in my model, it has again the same effect as a decrease in y.

[46] Faberman and Menzio (2017) find a negative relationship between wages and applications in the EOPP data, but attribute this result to the fact that they cannot control for all relevant heterogeneity. See also the discussion in Marinescu and Wolthoff (2015).

Figure 4: Equilibrium Outcomes.



trade also end up hiring more productive workers, in the sense of first-order stochastic dominance; firm posting c_1^* or c_{15}^* hire workers who on average are respectively 1.0 or 1.5 standard deviations better than the median qualified applicant.

The bottom right plot shows workers' probability to receive a job offer conditional on drawing a productivity x. The plot confirms that applications to contracts with better terms of trade are less likely to result in a job offer than applications to contracts with worse terms of trade. More importantly, the plot reveals that the variation across terms of trade is relatively small compared to the variation caused by productivity differences. In other words, the main determinant of whether a worker's application is successful is his fit for the position.

4.4 Discussion

In this section, I discuss the insights generated by the calibrated model. For this, I perform a number of counterfactual exercises, the results of which are displayed in column II to VI of table 3, along with the baseline calibration results in column I for comparison.

Match Efficiency. A researcher using a Cobb-Douglas matching function assumes that firms' monthly job-filling rate equals $\Omega\left(\frac{v}{u}\right)^{-\zeta}$, where Ω denotes aggregate efficiency of the matching process and ζ is the elasticity of the matching function. Hence, the monthly job-filling probability satisfies $1-e^{-\Omega\left(\frac{v}{u}\right)^{-\zeta}}$. Setting $\zeta=0.5$, as in Davis et al. (2013), the data generated by my model then imply a procyclical match efficiency with an elasticity of 5.1. This is roughly in line with recent empirical results, such as those by Sedláček (2016) who finds that 40% of the volatility of unemployment is due to fluctuations in match efficiency.

Multiplicity. The multiplicity of equilibrium means that different outcomes can arise for given parameter values. To assess the quantitative importance of this, column II presents the effects of a shock to the equilibrium selection rule from $\gamma=1$ to $\gamma=0$. This worsening of the frictions in the market has a limited effect on certain outcomes, such as workers' search intensity and firms' recruiting intensity, which both increase only slightly. At the same time however, the shock substantially affects unemployment, which goes up by 1.4 percentage points to 9.1%.

Amplification. As pointed out by Hagedorn and Manovskii (2008), the value of h plays an important role for the degree of amplification created by search models, as measured by the volatility of labor market tightness with respect to productivity. To analyze how much of the amplification in my model comes from $h=0.888$, which corresponds to 84.1% of average match output, versus other ingredients, I perform two counterfactual exercises.

In column III, I recalibrate the model, but instead of targeting the elasticity of unemployment, I target h to be 95.5% of average match output, in line with Hagedorn and Manovskii (2008). Doing this causes the fraction of qualified applicants q to hit the upper bound of 1, which means that the remaining five moments cannot all be matched. Dropping the moment on screening time as a target yields $h=0.971$. In that case, the elasticity of labor market tightness implied by my model is 43.0, which is 90% higher than the corresponding value of 22.5 in the calibration of Hagedorn and Manovskii (2008). In column IV, I choose parameter values to make my model as similar to a standard DMP model as possible.[47] That is, workers apply once per period ($A=1$, $\alpha=0$), all workers are qualified ($q=1$) and the productivity distribution is degenerate at 1 ($\sigma=0$), such that firms have no incentive to invest in recruitment ($r=0$). Fixing h to 84.1% of match output leaves only one parameter, i.e. the cost of entry k_V, which typically is calibrated by targeting workers' job-finding probability of 0.412.[48] This results in an elasticity of labor market tightness equal to 4.2, which is less than 40% of the value generated by the baseline calibration.

Summarizing, these results indicate that my model generates roughly 2 times as much amplification as a standard model. The reason is straightforward: unlike the standard model, my model features choices along the intensive margin for both workers and firms. As discussed above, these

[47]However, the model in Hagedorn and Manovskii (2008) is not nested, because I use urn-ball rather than Cobb-Douglas and impose the Hosios (1990) condition by using a directed search model.

[48]Choosing k_V to match firms' job-filling probability yields similar results, because the queue that solves $\Psi=\left(1-e^{-\lambda}\right)/\lambda=0.412$ implies a job-filling probability of 0.883, which is nearly identical to the empirical counterpart. Neither calibration is of course able to explain the data on the average number of applicants and interviews.

Table 3: Counterfactual Exercises.

	(I) baseline	(II) $\gamma = 0$ (shock)	(III) amplification (re-cal.)	(IV) (re-cal.)	(V) $k_R = 0$ (re-cal.)	(VI) high k_R (shock)
Parameter values						
Max. no. of applications A	15	15	15	1	15	15
Entry cost k_V	0.394	0.394	0.175	–	0.300	0.394
Recruiting cost k_R	0.008	0.008	0.013	–	0	0.179
Search cost k_A	0.041	0.041	0.009	–	0.042	0.041
Payoff from unempl. h	0.888	0.888	0.971	0.841	0.890	0.888
Fraction of qualified appl. q	0.690	0.690	1.000	1	0.576	0.690
Shape parameter σ	0.048	0.048	0.016	0	0.042	0.048
Selection rule γ	1	0	1	–	-	1
Equilibrium outcomes						
Unemployment	0.077	0.091	0.077	0.077	0.077	0.089
Search intensity	3.44	3.52	3.48	0	3.38	1.66
Avg. recruiting intensity	13.95	15.15	6.11	0	∞	0.568
Job-finding probability	0.412	0.371	0.412	0.412	0.412	0.376
Job-filling probability	0.884	0.901	0.884	0.883	0.884	0.711
Avg. match output	1.056	1.059	1.017	1	1.050	1.024
Elasticities						
Search intensity	10.2	10.1	45.1	0	9.1	27.0
Avg. recruiting intensity	1.2	0.8	6.8	0	0	12.9
Labor market tightness	11.3	11.2	43.0	4.2	11.5	15.2

choices generate a matching process which is more efficient (for a given tightness) during booms than during recessions, amplifying the effect of a productivity shock.[49]

No Recruiting Cost. Given the small value of k_R, a natural question is what results would be obtained if one calibrated the model without choice of recruiting intensity, by exogenously imposing $k_R = 0$. Again dropping the moment on screening time as a target, this yields the values in column V. While some outcomes are similar to the baseline calibration, other results differ. For example, the fraction of qualified applicants q drops to 0.576, as it is now the only instrument to explain firms' limited number of interviews. Further, k_V decreases to 0.300. Perhaps most importantly, by proposition 6, this calibration of course fails to reveal the multiplicity of equilibrium.

High Recruiting Cost. As mentioned in the introduction, the fact that firms do not spend much time on recruiting in the EOPP data can be the result of either a high effort at low cost, or a low effort at high cost. The calibration reveals that the former explanation is the correct one. To understand what the data should have looked like to obtain the alternative result, I consider a counterfactual economy in which the recruiting cost k_R is higher, but firms in equilibrium spend the same resources on recruiting in total. Keeping other parameters at their calibrated values, this requires a recruiting cost k_R of 0.179. This magnifies the frictions substantially, as displayed in column VI of table 3. The high recruiting cost causes firms to choose a very low recruiting intensity; they interview only 1.38 of their 5.03 applicants on average. Despite the lower hiring standards that they set in response (0.961), their job-filling probability drops to 0.711. The ineffectiveness of the recruiting process causes workers to send fewer applications (2.66) and reduces their job-finding probability to 0.376, increasing unemployment to 8.9%. Finally, workers' search intensity and firms' recruiting intensity both become much more elastic.

5 Conclusion

Job applicants generally differ in numerous dimensions, including their productivity and their outside options. As a result, firms' recruiting decisions tend to be far from trivial. In this paper, I present a search model to analyze these decisions. In the model, workers choose how many simultaneous applications they wish to send, which creates ex post heterogeneity in the probability with which they accept job offers. In addition, workers differ in their match productivity. To deal with this heterogeneity, firms post contracts that include a recruiting strategy, in addition to terms of trade. In particular, firms post a recruiting intensity, determining how many applicants they interview, and a hiring standard, describing which applicants are sufficiently productive to be hired. An interview is valuable because it reveals the productivity of the applicant. Hence, a higher recruiting intensity allows the firm to rank more applicants and form a better match, but a higher cost, since each

[49] See also the discussion in Kaas (2010) who similarly obtains amplification through endogenous search intensity in a random search model.

interview takes time or resources away from production. Similarly, a more selective hiring standard leads to better matches, but at a cost of remaining unmatched for longer.

I characterize equilibrium and find that outcomes like uniqueness of equilibrium and cyclicality of firms' recruiting intensity crucially depend on workers' search cost and firms' recruiting cost. To shed some light on these effects, I demonstrate how the model can be calibrated using the EOPP data. The results of this exercise indicate that a continuum of equilibria exists. Hiring standards are countercyclical, while—for the calibrated economy—recruiting intensity is procyclical. Finally, I show that my model creates twice as much amplification as a standard model and gives rise to a matching process which appears to be more efficient in booms to a researcher who looks through the lens of a standard Cobb-Douglas matching function.

Various extensions appear to be promising avenues for future research. First, allowing firms to create multiple vacancies would not only make the model more realistic but could also generate important interactions with firms' recruiting intensity, as it provides an alternative way for firms to hire with larger probability.[50] Second, ex ante heterogeneity in workers' productivity would endogenize the productivity distribution that each firm faces, creating an interesting tradeoff between ex ante sorting and ex post screening.[51] Third, a natural alternative to the simultaneous approach explored here would be to allow firms to interview applicants sequentially as in Lester and Wolthoff (2016) for $A = 1$; this would imply that job-offer probabilities are flat above some endogenous cutoff instead of increasing in productivity everywhere, but the implications for other results remain an open question. Finally, the framework presented in this paper might be a useful tool for analyzing the matching process beyond the specific sample of the EOPP data. As mentioned, data from various employment websites has become available in recent years, which strongly supports the idea that workers send multiple applications simultaneously. Data on the firm side of the process has historically been more limited, but firms' increasing use of *applicant tracking systems* will likely provide a wealth of new information.[52]

[50] Davis et al. (2013) document large heterogeneity in the number of vacancies and the job-filling probabilities across firms. See Kaas and Kircher (2015) for a model of these observations.

[51] See Eeckhout and Kircher (2010) and Cai et al. (2016, 2017) for some first work in this area, although all these papers restrict attention to $A = 1$. Gautier and Wolthoff (2009) analyze heterogeneous workers each sending two applications, but exogenously impose one interview per firm.

[52] Weber (2012) provides an informal description of these systems. See Burks et al. (2015) for a study that uses detailed applicant data for a number of firms. Finally, note also that some countries have representative survey data with detailed information on firms' recruiting behavior; see Carrillo-Tudela et al. (2016) for an analysis of the German IAB Job Vacancy Survey.

A Omitted Definitions

Queue Length. For the perturbed economy, let $G_{i:a,\varepsilon}$ represent the marginal distribution of a worker's i-th out of a applications. We then specify $\lambda(c) = \lim_{\varepsilon \to 0} \lambda_\varepsilon(c)$, where $\lambda_\varepsilon(c)$ is defined by

$$\int_{(0,\underline{x}] \times [0,r] \times [0,w]} \lambda_\varepsilon(\widetilde{c})\, d\left[vF_\varepsilon(\widetilde{c})\right] = s \int_0^\infty \sum_{a=1}^{A} p_a(\alpha) \sum_{i=1}^{a} G_{i:a,\varepsilon}(c)\, dH_\varepsilon(\alpha) \;\; \forall c \in \mathcal{C}. \tag{14}$$

The right-hand side denotes the total mass of applications sent to contracts with recruiting intensities no higher than r, hiring standards no higher than $\underline{x}$, and terms of trade no higher than w. The left-hand side integrates queue lengths across all firms posting such contracts, yielding the total mass of applications received by these firms. Both masses need to be the same for each possible $c = (\underline{x}, r, w)$.[53]

Acceptance Probability. Again for the perturbed economy, let $\xi_{i:a,\varepsilon}(\mathbf{c}_a)$ be the counterpart of $\xi_{i:a}(\mathbf{c}_a)$, let $\overline{p}_{i:a,\varepsilon}(c)$ be the conditional probability that an arbitrary applicant to a firm posting c applied with his i-th out of a applications, and let $\overline{G}_{-i:a,\varepsilon}(\mathbf{c}_{-i:a}; c)$ denote the conditional distribution of his remaining applications $\mathbf{c}_{-i:a} = (c_{1:a}, \ldots, c_{i-1:a}, c_{i+1:a}, \ldots, c_{a:a})$. We then specify $\theta(c) = \lim_{\varepsilon \to 0} \theta_\varepsilon(c)$, where $\theta_\varepsilon(c)$ is defined by[54]

$$\theta_\varepsilon(c) = \sum_{a=1}^{A} \sum_{i=1}^{a} \overline{p}_{i:a,\varepsilon}(c) \int_{\mathcal{C}^{a-1}} \xi_{i:a,\varepsilon}(\mathbf{c}_a)\, d\overline{G}_{-i:a,\varepsilon}(\mathbf{c}_{-i:a}; c). \tag{15}$$

Job-Finding Probability. The equilibrium job-finding probability of a worker equals

$$\Psi^* = 1 - \int_0^\infty \int_{\mathcal{C}^a} \sum_{a=1}^{A} p_a(\alpha) \prod_{i=1}^{a} \left(1 - \psi(c_{i:a})\right)\, dG_a^*(\mathbf{c}_a)\, dH^*(\alpha),$$

where the second term is the expectation (over $\mathbf{c}_a$, a, and α) of the probability that all applications sent by a worker fail to result in a job offer.

[53] See Kircher (2009) for a detailed discussion of a similar expression.

[54] See Galenianos and Kircher (2009) for a detailed discussion of a similar expression.

B Proofs

B.1 Proof of Proposition 1

Consider a firm with recruiting intensity $r \geq 0$ and a queue $q\lambda > 0$ of qualified applicants, and suppose that a qualified applicant has a certain characteristic with probability ϕ.[55] I will first derive the probability that the firm interviews at least 1 applicant with the characteristic and then use that to prove the lemma. To simplify notation, define $\ell = q\lambda$ and $\rho = \frac{r}{r+1}$.

Number of Interviews. Given a number of qualified applicants $n_Q \sim \text{Poi}(\ell)$ and a potential number of interviews $n_R \sim \text{Geo}(\rho)$, the actual number of interviews is $n_I = \min\{n_Q, n_R\}$. This equals 0 with probability $\mathbb{P}[\min\{n_Q, n_R\} = 0] = e^{-\ell}$ and $n \in \mathbb{N}_1$ with probability

$$\begin{aligned}
\mathbb{P}[\min\{n_Q, n_R\} = n] &= \sum_{i=n+1}^{\infty} e^{-\ell}\frac{\ell^i}{i!}\rho^{n-1}\frac{1}{r+1} + e^{-\ell}\frac{\ell^n}{n!}\sum_{i=n}^{\infty}\rho^{i-1}\frac{1}{r+1} \\
&= \rho^{n-1}\left(e^{-\ell}\frac{\ell^n}{n!} + \frac{1}{r+1}\sum_{i=n+1}^{\infty} e^{-\ell}\frac{\ell^i}{i!}\right).
\end{aligned}$$

Expectation. Conditional on n interviews, the probability to meet at least 1 applicant with the characteristic is $1 - (1-\phi)^n$. Taking the expectation over n yields

$$\begin{aligned}
\sum_{n=0}^{\infty}[1 - (1-\phi)^n]\,\mathbb{P}[\min\{n_Q, n_R\} = n] &= \sum_{n=1}^{\infty}\mathbb{P}[\min\{n_Q, n_R\} = n] \\
&\quad -\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}e^{-\ell}\frac{\ell^n}{n!} \\
&\quad -\frac{1}{r+1}\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}\sum_{i=n+1}^{\infty}e^{-\ell}\frac{\ell^i}{i!}.
\end{aligned}$$

Consider each of the three terms on the right-hand side separately. The first term simplifies to $1 - e^{-\ell}$. The second term equals

$$\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}e^{-\ell}\frac{\ell^n}{n!} = \frac{1}{\rho}\left(e^{-\ell(1-\rho+\rho\phi)} - e^{-\ell}\right).$$

Finally, apply a change in the order of summation in the third term to get

$$\frac{1}{r+1}\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}\sum_{i=n+1}^{\infty}e^{-\ell}\frac{\ell^i}{i!} = \frac{1}{r}\left[\frac{\rho(1-\phi) - e^{-\ell(1-\rho+\rho\phi)}}{1-\rho+\rho\phi} + e^{-\ell}\right].$$

[55] In this proof, I omit arguments as much as possible to keep notation simple.

Combining the three terms gives, after some simplification,

$$\frac{\phi}{1-\rho+\rho\phi}\left(1-e^{-\ell(1-\rho+\rho\phi)}\right). \tag{16}$$

Hiring Probability. Consider now the original problem. To hire a worker with productivity greater than x, satisfying $x \geq \underline{x}$, a firm needs to interview at least one such applicant who, in addition, must accept the job offer. That is, $\phi = \theta\left(1-Q\left(x\right)\right) = \frac{\mu(x;\lambda,\theta)}{q\lambda}$. Substituting this into (16) yields (3). Workers with $x < \underline{x}$ will never be hired, hence $\eta\left(x;\cdot\right) = \eta\left(\underline{x};\cdot\right)$ for those productivity draws.

Job-Offer Probability. A firm hires a worker with probability $\eta\left(\underline{x};\cdot\right)$. Since job offers are accepted with probability θ, this requires $\frac{\eta(\underline{x};\cdot)}{\theta}$ job offers in expectation. Each of the λ applicants is ex ante equally likely to receive one of these job offers. Hence $\psi = \frac{\eta(\underline{x};\cdot)}{\lambda\theta}$. Substitution of (3) yields the desired expression. □

B.2 Proof of Lemma 1

Monotonicity. Using subscripts to indicate partial derivatives and omitting arguments to keep notation simple, a straightforward calculation yields

$$\eta_x = -\frac{\left(1-e^{-\kappa}\right)q\lambda + r\mu\kappa e^{-\kappa}}{\left(r+1\right)\kappa^2}q\lambda\theta Q' < 0, \tag{17}$$

$$\eta_\lambda = \frac{\mu e^{-\kappa}}{\lambda} > 0,$$

and

$$\eta_r = \frac{\left(q\lambda-\mu\right)\mu\left(1-e^{-\kappa}-\kappa e^{-\kappa}\right)}{\left(r+1\right)^2\kappa^2} > 0.$$

Hence, η is strictly decreasing in x and strictly increasing in λ and r.

Concavity. The second-order partial derivative of η with respect to λ equals

$$\eta_{\lambda\lambda} = -\frac{\mu\kappa}{\lambda^2}e^{-\kappa} < 0,$$

while the determinant of the Hessian with respect to λ and r equals

$$2\left(\frac{\mu}{r+1}\right)^3\frac{q\lambda-\mu}{\lambda^2\kappa^2}e^{-\kappa}\left(1-e^{-\kappa}-\kappa e^{-\kappa}\right) > 0.$$

Hence, η is strictly concave in λ and r.

Supermodularity. The cross-partial of η is equal to

$$\eta_{r\lambda} = \frac{(q\lambda - \mu)\,\mu e^{-\kappa}}{\lambda\,(r+1)^2} > 0.$$

Hence, η is strictly supermodular in λ and r. $\square$

B.3 Proof of Lemma 2

Given a number of applications a, a worker chooses a set of contracts $\mathbf{c}_a = (c_1, \ldots, c_a)$ to solve

$$U_a = \max_{\mathbf{c}_a} U_0 + \sum_{i=1}^{a} \xi_{i:a}(\mathbf{c}_a)\,\psi(c_i)\,(w_i - U_0)\,. \tag{18}$$

Note that equation (8) implies

$$\xi_{i:a}(\mathbf{c}_a) = (1 - \psi(c_a))\,\xi_{i:a-1}(\mathbf{c}_{a-1})\,,$$

for all $i \in \{1, \ldots, a-1\}$. Substituting this into (18) and rearranging the result yields

$$\begin{aligned} U_a &= \max_{c_a} \psi(c_a)\,w_a + (1-\psi(c_a)) \max_{\mathbf{c}_{a-1}} \left(U_0 + \sum_{i=1}^{a-1} \xi_{i:a-1}(\mathbf{c}_{a-1})\,\psi(c_i)\,(w_i - U_0) \right) \\ &= \max_{c} \psi(c)\,w + (1-\psi(c))\,U_{a-1}, \end{aligned}$$

which is the desired recursive formulation. $\square$

B.4 Proof of Proposition 2

Notation. For a given U_0 and U_1, define Δ_1 as the marginal gain in workers' payoff as a result of sending the (first) application. That is, $\Delta_1 \equiv U_1 - U_0$.

Optimal Hiring Standard. Inspection of (11) immediately reveals that firms' optimal hiring standard $\underline{x}_1$ satisfies $M(\underline{x}_1) = U_0$. That is, a firm will only form matches for which the corresponding match surplus $M(x) - U_0$ is non-negative. Using (5) and (10), this implies

$$\underline{x}(\Delta_1) = h - y + \beta(1-\delta)\,\Delta_1, \tag{19}$$

which is independent of the firm's choice of r and λ.

Optimal Queue and Recruiting Intensity. Evaluating (11) in (19), followed by integration by parts and substitution of (10), gives

$$\max_{r,\lambda} \frac{1}{1-\beta(1-\delta)} \int_{\underline{x}(\Delta_1)}^{\infty} \eta(x; r, \lambda, 1)\; dx - \lambda\Delta_1 - k_V - k_R r. \tag{20}$$

By lemma 1 and the linearity of integration, equation (20) is a strictly concave function of r and λ. Combined with the fact that the firm's payoff tends to $-\infty$ for $r \to \infty$ or $\lambda \to \infty$, strict concavity implies that a unique solution (r, λ) exists for any Δ_1. Using subscripts to indicate partial derivatives of η, this solution must satisfy the first-order condition with respect to λ,

$$\frac{1}{1-\beta(1-\delta)} \int_{\underline{x}(\Delta_1)}^{\infty} \eta_\lambda(x; r, \lambda, 1)\, dx - \Delta_1 = 0, \tag{21}$$

and either the the boundary point $r = 0$ or the first-order condition with respect to r,

$$\frac{1}{1-\beta(1-\delta)} \int_{\underline{x}(\Delta_1)}^{\infty} \eta_r(x; r, \lambda, 1)\, dx - k_R = 0. \tag{22}$$

Equilibrium. The above derivations pin down λ and r as functions of Δ_1. Evaluating (20) in these functions gives the value of a vacancy V as function of Δ_1 alone. Note that V is strictly decreasing in Δ_1, as

$$\frac{dV}{d\Delta_1} = -\frac{\beta(1-\delta)}{1-\beta(1-\delta)} \eta(\underline{x}; r, \lambda, 1) - \lambda < 0, \tag{23}$$

by the envelope theorem.[56] Hence, a unique solution Δ_1^* to the free entry condition $V = 0$ exists. This solution subsequently implies unique equilibrium values for $U_0^* = \frac{h+\beta\Delta_1^*}{1-\beta}$, $U_1^* = \frac{h+\Delta_1^*}{1-\beta}$, the hiring standard $\underline{x}_1^*$, the queue λ_1^*, and the recruiting intensity r_1^*.

Given these variables, the equilibrium terms of trade w_1^* follow directly from the constraint (9). Workers' job-finding probability equals $\Psi^* = \psi(c_1^*)$, where $c_1^* = (\underline{x}_1^*, r_1^*, w_1^*)$, and the measure of searchers s^* is determined by the steady state condition, i.e.

$$s^* = \frac{\delta}{\delta + (1-\delta)\Psi^*}.$$

Finally, the equilibrium measure of firms entering the market satisfies $v^* = \frac{s^*}{\lambda_1^*}$. $\square$

B.5 Proof of Proposition 3

Hiring Standard. First, consider firms' hiring standards. Borrowing the notation $\Delta_1 = U_1 - U_0$ from the proof of proposition 2, the implicit function theorem applied to the free entry condition $V = 0$ implies

$$\frac{d\Delta_1^*}{dy} = \left. \frac{\eta(\underline{x}; r, \lambda, 1)}{\beta(1-\delta)\eta(\underline{x}; r, \lambda, 1) + (1-\beta(1-\delta))\lambda} \right|_{\underline{x}_1^*, r_1^*, \lambda_1^*} > 0. \tag{24}$$

Equation (19) then yields

$$\frac{d\underline{x}_1^*}{dy} = -\left. \frac{(1-\beta(1-\delta))\lambda}{\beta(1-\delta)\eta(\underline{x}; r, \lambda, 1) + (1-\beta(1-\delta))\lambda} \right|_{\underline{x}_1^*, r_1^*, \lambda_1^*} < 0. \tag{25}$$

Hence, firms relax their hiring standard in response to the aggregate productivity shock.

[56] If the boundary condition $r = 0$ binds, then $\frac{dr}{d\Delta_1} = 0$ and $\frac{dV}{d\Delta_1}$ continues to be negative.

Recruiting Intensity. Next, consider firms' recruiting intensity. Define

$$Z\left(\underline{x},r,\lambda\right)=\int_{\underline{x}}^{\infty}\eta\left(x;r,\lambda,1\right)\,dx$$

to simplify notation. Using subscripts to indicate partial derivatives of η and Z, total differentiation of the first-order conditions (21) and (22) gives

$$\frac{1}{1-\beta\left(1-\delta\right)}\left[Z_{\lambda\lambda}\left(\underline{x},r,\lambda\right)\frac{d\lambda}{dy}+Z_{r\lambda}\left(\underline{x},r,\lambda\right)\frac{dr}{dy}-\eta_{\lambda}\left(\underline{x};r,\lambda,1\right)\frac{d\underline{x}}{dy}\right]-\frac{d\Delta_1}{dy}=0$$

and

$$\frac{1}{1-\beta\left(1-\delta\right)}\left[Z_{r\lambda}\left(\underline{x},r,\lambda\right)\frac{d\lambda}{dy}+Z_{rr}\left(\underline{x},r,\lambda\right)\frac{dr}{dy}-\eta_{r}\left(\underline{x};r,\lambda,1\right)\frac{d\underline{x}}{dy}\right]=0. \tag{26}$$

Solving for $\frac{dr}{dy}$, evaluating in equilibrium, and substituting (24) and (25) yields

$$\frac{dr_1^*}{dy}=\left.\frac{\left[\lambda\eta_{\lambda}\left(\underline{x};r,\lambda,1\right)-\eta\left(\underline{x};r,\lambda,1\right)\right]Z_{r\lambda}\left(\underline{x},r,\lambda\right)-\lambda\eta_{r}\left(\underline{x};r,\lambda,1\right)Z_{\lambda\lambda}\left(\underline{x},r,\lambda\right)}{\left[Z_{\lambda\lambda}\left(\underline{x},r,\lambda\right)Z_{rr}\left(\underline{x},r,\lambda\right)-Z_{r\lambda}^2\left(\underline{x},r,\lambda\right)\right]\left[\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\eta\left(\underline{x};r,\lambda,1\right)+\lambda\right]}\right|_{\underline{x}_1^*,r_1^*,\lambda_1^*}.$$

By lemma 1, the denominator of this expression is positive. Substituting the derivates of η provided in the proof of that lemma reveals that the numerator of $\frac{dr_1^*}{dy}$ has the same sign as

$$\left.\int_{\underline{x}}^{\infty}\left(\frac{\lambda q-\mu\left(\underline{x};\lambda,1\right)}{\kappa\left(\underline{x};r,\lambda,1\right)}-\frac{\lambda q-\mu\left(x;\lambda,1\right)}{\kappa\left(x;r,\lambda,1\right)}\right)\mu\left(x;\lambda,1\right)\kappa\left(x;r,\lambda,1\right)e^{-\kappa\left(x;r,\lambda,1\right)}\,dx\right|_{\underline{x}_1^*,r_1^*,\lambda_1^*}.$$

This expression is negative since

$$\frac{\partial}{\partial x}\frac{\lambda q-\mu\left(x;\lambda,1\right)}{\kappa\left(x;r,\lambda,1\right)}>0.$$

Hence, $\frac{dr_1^*}{dy}<0$, i.e. equilibrium recruiting intensity moves counter to aggregate productivity.

Queue Length. Evaluated in equilibrium, equation (26) implies

$$\frac{d\lambda_1^*}{dy}=\left.\frac{1}{Z_{r\lambda}\left(\underline{x},r,\lambda\right)}\left(\eta_{r}\left(\underline{x};r,\lambda,1\right)\frac{d\underline{x}}{dy}-Z_{rr}\left(\underline{x},r,\lambda\right)\frac{dr}{dy}\right)\right|_{\underline{x}_1^*,r_1^*,\lambda_1^*}.$$

By lemma 1 and the results above, this expression is negative, i.e. the equilibrium queue length goes down in response to the increase in aggregate productivity. □

B.6 Proof of Proposition 4

Hiring Standard. Equations (17) reveals that $\frac{\partial\eta_1^*}{\partial x}\to 0$ if $Q'\left(\underline{x}_1^*\right)\to 0$, while (25) implies that $\frac{d\underline{x}_1^*}{dy}$ does not depend on $Q'\left(\underline{x}_1^*\right)$. Hence, the result follows.

Recruiting Intensity. If $k_R \to 0$, then equation (22) implies that $r_1^* \to \infty$ for any y, $\underline{x}_1^*$ and λ_1^*. In contrast, if $k_R \to \infty$, then the left-hand side of (22) is always negative and the boundary solution $r = 0$ arises for any y, $\underline{x}_1^*$ and λ_1^*. In either case, $\frac{dr_1^*}{dy} = 0$. Because $\frac{\partial \eta_1^*}{\partial r} < \infty$, the result follows.

B.7 Proof of Lemma 3

Notation. Given $U_0, \dots, U_A$, define Δ_i as the marginal gain in workers' payoffs as a result of application number $i \in \{1, \dots, A\}$. That is, $\Delta_i \equiv U_i - U_{i-1}$.

Ordering of Queue Lengths. I start by showing that the indifference curves of applicants implied by (9) pin down queue lengths for each contract. Given $U_1, \dots, U_A$ and acceptance probabilities $\theta(c)$, define $\lambda_i(c) > 0$ as the queue length that solves[57]

$$\Delta_i = \psi(\underline{x}, r, \lambda(c), \theta(c))(w - U_{i-1}), \tag{27}$$

As workers' job-offer probabilities are strictly decreasing in λ, a unique solution exists for each i and c. Denote the corresponding job-offer probability by $\psi_i(c)$. As applicants are willing to apply to a firm until (27) binds, a firm posting a contract c expects to attract a queue $\lambda(c) = \max_i \{\lambda_i(c)\}$ with corresponding job-offer probability $\psi(c) = \min_i \{\psi_i(c)\}$.

Single Crossing. Differentiation of (9) reveals that each $\psi_i(c)$ is strictly decreasing in w. Moreover, the slope is more negative for larger i, because U_{i-1} is increasing in i. This implies a single-crossing condition: if $\psi_i(c) = \psi_{i+1}(c)$ for some $c = (\underline{x}, r, w)$, then $\psi_i(\widetilde{c}) > \psi_{i+1}(\widetilde{c})$ for all $\widetilde{c} = (\underline{x}, r, \widetilde{w})$ satisfying $\widetilde{w} > w$. Figure 2 illustrates this for $A = 2$.

Application Regions. We can then define subsets $\mathcal{C}_i$ of the contract space, consisting of the contracts for which $\psi_i(c)$ is strictly smaller than all other $\psi_j(c)$, i.e.

$$\mathcal{C}_i = \left\{ c \in \mathcal{C} | \psi(c) < \min_{j \neq i} \{\psi_j(c)\} \right\}.$$

These subsets have a number of useful properties. First, by construction, if there exist contracts in $\mathcal{C}_i$ which are part of an equilibrium, they can only receive the i-th application of workers. Second, since a worker's payoff only depends on r and $\underline{x}$ through $\psi(c)$, each $\mathcal{C}_i$ can be written as $\mathcal{X} \times \mathcal{R} \times \mathcal{W}_i$, where $\mathcal{W}_i$ is an appropriately defined subset of $\mathcal{W}$. Third, the single-crossing condition implies that each $\mathcal{W}_i$ is a open interval, which can be denoted by $(\overline{w}_{i-1}, \overline{w}_i)$, where $\overline{w}_0$ is the lowest value for the terms of trade that would receive applications, $\overline{w}_i$ is the highest terms of trade that provides workers applying i times with a payoff U_i, and $\overline{w}_A = \infty$.

[57] Contracts for which the queue is zero can never be part of an equilibrium and will therefore be ignored in the remainder of the proof.

Application Behavior. By construction, a worker must now send his i-th application to contracts in the closure of $\mathcal{C}_i$, i.e. $\mathcal{X} \times \mathcal{R} \times [\overline{w}_{i-1}, \overline{w}_i]$. After all, a contract $c = (\underline{x}, r, w)$ that offers $w > \overline{w}_i$ or $w < \overline{w}_{i-1}$ attracts a queue $\lambda(c) > \lambda_i(c)$, implying a job-offer probability $\psi(c) < \psi_i(c)$. Such a contract would provide the worker with a marginal payoff that is strictly lower than Δ_i, which violates the equilibrium conditions.

Acceptance Probability. Firms and workers need to form beliefs regarding the probability $\theta(c)$ that a job offer from a firm posting c will get accepted. While it is straightforward to derive this probability for the equilibrium contracts, its value off the equilibrium path requires a more careful analysis. Galenianos and Kircher (2009) analyze this issue in detail and prove formally using the trembles in firms' behavior described in section 3.1 that the acceptance probability is constant within each $\mathcal{W}_i$. Although their model does not include hiring standards and recruiting intensities, their proof trivially extends to all contracts in $\mathcal{C}_i$, as acceptance decisions are independent of $\underline{x}$ and r. Hence, $\theta(c) = \theta_i$ for all $c \in \mathcal{C}_i$.

Further, if a contract $(\underline{x}, r, \overline{w}_i)$ at the boundary between $\mathcal{C}_i$ and $\mathcal{C}_{i+1}$ is part of an equilibrium, then $\theta(\underline{x}, r, \overline{w}_i)$ must equal θ_{i+1}, or else a deviation to $\widetilde{c} = (\underline{x}, r, \overline{w}_i + \varepsilon)$ would be profitable by providing the firm a discretely higher job-filling probability. To see this, note that a firm's job-filling probability can be written as

$$\eta(\underline{x}; c) = \frac{\theta(c)(1 - Q(\underline{x}))}{\frac{1}{r+1} + \frac{r}{r+1}\theta(c)(1 - Q(\underline{x}))} \left(1 - e^{-\kappa(\underline{x}; c)}\right).$$

Since $\psi(c)$ is continuous in $\overline{w}_i$, so is $\kappa(\underline{x}; c)$. Right-continuity of $\eta(\underline{x}; c)$ in $\overline{w}_i$ therefore requires right-continuity of $\theta(c)$ in this point.

Optimization Problem. The above results imply that equilibrium will give rise to at least one contract in each $\underline{\mathcal{C}}_i \equiv \mathcal{X} \times \mathcal{R} \times [\overline{w}_{i-1}, \overline{w}_i)$. A firm posting a contract in $\underline{\mathcal{C}}_i$ expects a queue determined by $\Delta_i = \psi(\underline{x}, r, \lambda_i(c), \theta_i)(w - U_{i-1})$. Solving the constraint for w and substituting the result into the firm's optimization problem yields, after some simplification,

$$\max_{c \in \underline{\mathcal{C}}_i} - \int_{\underline{x}}^{\infty} (M(x) - U_{i-1})\, d\eta(x; r, \lambda_i(c), \theta_i) - \lambda_i(c)\theta_i \Delta_i - k_V - k_R r. \tag{28}$$

As for $A = 1$, the firm's payoff only depends on $w \in [\overline{w}_{i-1}, \overline{w}_i)$ through the queue length $\lambda_i(c)$, such that we can analyze the firm's optimization problem in terms of $(\underline{x}, r, \lambda)$, instead of $(\underline{x}, r, w)$.

Optimal Hiring Standard. Equation (28) immediately reveals that the optimal hiring standard $\underline{x}_i$ of a firm posting a contract in $\underline{\mathcal{C}}_i$ must solve $M(\underline{x}_i) = U_{i-1}$. Using (10), this implies

$$\underline{x}_i(U_0, U_{i-1}) = \delta h - y + (1 - \beta(1 - \delta)) U_{i-1} - \delta U_0. \tag{29}$$

which is increasing in i and independent of the firm's choice of w and r.

Optimal Queue and Recruiting Intensity. Evaluating (28) in $\underline{x}_i(U_0, U_{i-1})$, followed by integration by parts and substitution of (10), gives

$$\frac{1}{1-\beta(1-\delta)}\int_{\underline{x}_i(U_0,U_{i-1})}^{\infty} \eta(x; r, \lambda, \theta_i)\ dx - \lambda\theta_i\Delta_i - k_V - k_R r. \tag{30}$$

By lemma 1, this expression is strictly concave in r and λ, implying that exactly one contract will arise in each $\underline{\mathcal{C}}_i$ in equilibrium. □

B.8 Proof of Proposition 5

I prove existence of the proposed equilibrium using Brouwer's fixed point theorem. To do so, take a measure of unmatched workers s and values $U_0, \ldots U_A$, and consider how the model structure and agents' decisions map them into new values s', $U_0', \ldots U_A'$.

First, unmatched workers choose their search intensity α to maximize $\sum_{a=1}^{A} p_a(\alpha) U_a - k_A\alpha$. The first term can be rewritten as[58]

$$\sum_{a=1}^{A} p_a(\alpha) U_a = U_0 + \sum_{a=1}^{A} \pi_a(\alpha)(\Delta_a - \Delta_{a+1}),$$

where $\Delta_a = U_a - U_{a-1}$, $\Delta_{A+1} = 0$ and $\pi_a(\alpha) \equiv \sum_{i=1}^{a}\sum_{j=i}^{A} p_j(\alpha)$ is a strictly concave function. Because $\Delta_a > \Delta_{a+1}$, concavity of $\pi_a(\alpha)$ implies concavity of the optimization problem in α. Hence, optimal search intensity is uniquely determined by the first-order condition $\sum_{a=1}^{A} p_a'(\alpha) U_a - k_A = 0$.

Second, consider $\underline{\mathcal{C}}_A$, for which $\theta_A = 1$. The values $U_0, \ldots, U_A$, equation (29) and the optimization problem (30) define $\underline{x}_A$, r_A and λ_A. These values imply a job-offer probability $\psi_A \equiv \psi(\underline{x}_A, r_A, \lambda_A, \theta_A)$. The solution for the other submarkets can recursively be obtained in a similar fashion, accounting for the fact that the acceptance probability in $\underline{\mathcal{C}}_i$ equals

$$\theta_i = \frac{\sum_{a=i}^{A} p_a(\alpha)\,\xi_{i:a}}{\sum_{a=i}^{A} p_a(\alpha)}, \tag{31}$$

where $\psi_i \equiv \psi(\underline{x}_i, r_i, \lambda_i, \theta_i)$ and where, with a slight abuse of notation, $\xi_{i:a}$ represents the probability $\prod_{j=i+1}^{a}(1-\psi_j)$ that a worker sending his i-th out of a applications to this segment will accept a job offer.

The updated values $U_1', \ldots, U_A'$ then follow from evaluating the free entry condition $V = 0$ in each of these solutions. The new value of unemployment U_0 follows from (5), which implies

$$U_0' = \frac{h}{1-\beta} + \frac{\beta}{1-\beta}\left(\sum_{a=1}^{A} p_a(\alpha)\sum_{i=1}^{a}\Delta_i - k_A\alpha\right).$$

[58] I am grateful to Xiaoming Cai for suggesting this part of the proof.

Finally, the new number of searchers is given by

$$s' = (1 - \Psi)\, s + (1 - (1 - \Psi)\, s)\, \delta,$$

where $\Psi = 1 - \sum_{i=1}^{A} p_i(\alpha) \prod_{n=1}^{i} (1 - \psi_n)$.

Hence, the model structure and agents' decisions represent a function which maps $\{s, U_0, \dots, U_A\}$ into $\{s', U_0', \dots, U_A'\}$. The mapping is continuous and its domain and co-domain are identical and compact.[59] Hence, by Brouwer's fixed point theorem, an equilibrium exists. $\square$

B.9 Proof of Lemma 4

I prove the lemma in detail for the case in which there exists a single corner solution and subsequently explain that the logic for the case with multiple corner solutions is identical.

One Corner Solution. Suppose there exists an equilibrium for some $\gamma^* \in [0,1]$ in which $\widehat{w}_i < \overline{w}_{i-1} = w_i^*$ for exactly *one* value of $i \in \{2, \dots, A\}$. Consider the optimal application portfolios of workers in this case. It is straightforward to see that workers sending $a \in \{1, \dots, i-2, i, \dots, A\}$ applications have a unique solution to their portfolio problem, i.e. $(c_1^*, \dots, c_a^*)$. The only workers who experience indifference are the ones sending $i-1$ applications; they apply to $\left(c_1^*, \dots, c_{i-2}^*, c_{i-1}^*\right)$ with probability γ^* and $\left(c_1^*, \dots, c_{i-2}^*, c_i^*\right)$ with probability $1-\gamma^*$.

This equilibrium must satisfy the fixed-point problem described in the proof of proposition 5, adjusted for the fact that the acceptance probabilities $\theta_1, \dots, \theta_i$ will differ from (31) when $\gamma^* < 1$. In particular, firms posting c_i^* attract—in addition to everyone applying at least i times—a fraction $1-\gamma^*$ of the workers sending $i-1$ applications, who will accept a job offer with certainty. Borrowing notation from the proof of proposition 5, this implies

$$\theta_i = \frac{\sum_{a=i}^{A} p_a(\alpha)\, \xi_{i:a} + (1-\gamma)\, p_{i-1}(\alpha)}{\sum_{a=i}^{A} p_a(\alpha) + (1-\gamma)\, p_{i-1}(\alpha)},$$

Further, firms posting c_{i-1}^* attract all workers who send at least i applications and a fraction $1-\gamma^*$ of the workers sending $i-1$ applications, who will again accept a job offer with certainty. Therefore,

$$\theta_{i-1} = \frac{\sum_{a=i}^{A} p_a(\alpha)\, \xi_{i-1:a} + \gamma p_{i-1}(\alpha)}{\sum_{a=i}^{A} p_a(\alpha) + \gamma p_{i-1}(\alpha)}.$$

Finally, a firm posting c_j^* for $j \in \{1, \dots, i-2\}$ attracts all workers sending at least j applications, but the workers sending $i-1$ applications come in two varieties: a fraction γ applied to $\left(c_1^*, \dots, c_{i-2}^*, c_{i-1}^*\right)$ and accepts the job offer with probability $\xi_{j:i-1}$, while the remaining fraction $1-\gamma$ applied to

[59] Note that U_A is bounded above by the output of a never-ending match. The game is not well-defined for $s=0$, but we can bound the domain for s away from zero since firms would never recover their entry costs for $s=0$. See Galenianos and Kircher (2005) for a detailed discussion.

$\left(c_1^*, \ldots, c_{i-2}^*, c_i^*\right)$ and accepts the job offer with probability $\xi_{j:i-1} \frac{1-\psi_i}{1-\psi_{i-1}}$. That is,

$$\theta_j = \frac{\sum_{a=j}^{i-2} p_a(\alpha)\, \xi_{j:a} + \sum_{a=i}^{A} p_a(\alpha)\, \xi_{j:a} + \left[(1-\gamma)\frac{1-\psi_i}{1-\psi_{i-1}} + \gamma\right] p_{i-1}(\alpha)\, \xi_{j:i-1}}{\sum_{a=j}^{A} p_a(\alpha)}.$$

Consider now a change in the equilibrium selection rule. A different value of γ changes the function which maps $\{s, U_0, \ldots, U_A\}$ into $\{s', U_0', \ldots, U_A'\}$, but does so in a continuous way. Hence, there exists a $\gamma' \neq \gamma^*$ such that the solution to the fixed-point problem, which is again guaranteed to exist by Brouwer's fixed point theorem, still features $\widehat{w}_i < \overline{w}_{i-1} = w_i^*$ for any γ between γ' and γ^*, justifying beliefs $\gamma < 1$. In other words, a continuum of equilibria exists.

Multiple Corner Solutions. If there exists an equilibrium for some $\gamma^* \in [0,1]$ in which $\widehat{w}_i < \overline{w}_{i-1} = w_i^*$ for *multiple* values of $i \in \{2, \ldots, A\}$, multiple types of workers will experience indifference between different application portfolios and the calculation of the acceptance probabilities becomes increasingly complex. However, none of that changes the fact that the mapping between $\{s, U_0, \ldots, U_A\}$ and $\{s', U_0', \ldots, U_A'\}$ depends on γ in a continuous way. Hence, the continuity argument above continues to apply and a continuum of equilibria exists. $\square$

B.10 Proof of Proposition 6

Small Cost. If $k_R = 0$, then all firms choose a recruiting intensity $r \to \infty$, such that $\kappa(x;c) = \mu(x;c)$. Hence, workers' preferences now depend on w and $\mu(\underline{x};c)$ alone. Moreover, for a given $\underline{x}$, firms' payoffs can be expressed as a function of the same variables, because

$$\eta(x;c) = 1 - e^{-\mu(x;c)} = 1 - \exp\left(-\frac{1-Q(x)}{1-Q(\underline{x})}\mu(\underline{x};c)\right).$$

Hence, all firms now have the same iso-payoff curve in $(w, \mu(\underline{x};c))$-space. This curve is strictly increasing and convex. The kink in workers' preferences at $\overline{w}_i$ can therefore never be part of an equilibrium; firms could always obtain a higher payoff by offering either lower or higher terms of trade.[60] Hence, $w_i^* > \overline{w}_{i-1}$ for all $i \in \{2, \ldots, A\}$. The desired result for k_R positive but sufficiently small then follows from continuity.

Large Cost. If $k_R \to \infty$, then all firms choose a recruiting intensity $r = 0$. Homogeneity of the workers (i.e. $q \to 1$, $Q(x)$ degenerate, and $k_A \to 0$ such that $p_A(\alpha) \to 1$) then reduces the model to the dynamic version of Galenianos and Kircher (2009). Their proof of $w_i^* = \overline{w}_{i-1}$ for all $i \in \{2, \ldots, A\}$, as provided in proposition 6.1 of Galenianos and Kircher (2005), therefore directly applies. The desired result away from the limit then follows again from continuity.

[60]See also the discussion in Kircher (2009) who proves a closely related result in a simpler environment.

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